Renormalization-group theory has given an elegant explanation to the universality principle of critical phenomena. Additionally, renormalization-group theory has provided us with a large variety of calculational approaches to the statistical physics of systems exhibiting complex behaviors. One such approach is position-space renormalization-group theory and, in parallel, the construction of hierarchical lattices [1-3] on which d-dimensional systems are solved exactly and exhibit complex behaviors. After a pedagogical review of renormalization-group theory and hierarchical lattices, the following applications will be illustrated: (1) The conversion of first-order phase transitions to second order by quenched random bonds.[4] (2) The phase diagram of random-field spin glasses in d=3.[5] (3) The geometric and phase transition properties of scale-free small-world hierarchical lattices.[6]