PHYSICAL REVIEW B VOLUME 28, NUMBER 7 1 OCTOBER 1983

# Potts model in random fields

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(Received 9 February 1983)

The Potts model is generalized to include the effects of random fields. The model is solved by the mean-field approximation. The mean-field approximation is shown to be exact in one dimension in the infinite-state limit, which suggests the exactness of this approximation for the infinite-state model in any dimension. A simple domain-energy argument is shown to explain the behavior of the infinite-state model in the ground state.

## I. INTRODUCTION

The problem of spin systems in random magnetic fields has attracted much attention recently. From the theoretical side, the O($N$)-symmetric spin systems including the Ising model have been investigated by a number of methods, such as the renormalization group,[1] numerical evaluations,[2] mean-field approximations,[3] effective interface system methods,[4] and explicit solutions of the spherical model.[5] It has been shown, in general, that random fields reduce the stability of ferromagnetic ordering. In particular, in low-dimensional systems an infinitesimal amount of random fields is considered to be enough to destroy the ferromagnetic ordering.[4] Experimental realizations of random-field models have been proposed and experiments have been carried out to test theoretical predictions.[6] However, there still remains some controversy, especially as to the lower critical dimensionality of the Ising case.[4] In this paper we propose a Potts-model version of the random-field problem. The model is solved by the mean-field approximation. Although it is, in general, impossible to solve this model exactly in an arbitrary dimensional space, it is suggested that the mean-field approximation may be exact in any dimension if the number of states $q$ is infinite. This was the case for the nonrandom Potts model.[7] We argue, on the basis of the explicit solution in one dimension, that the exactness of the mean-field approximation may hold even in the presence of random fields. The phase diagram of the infinite-state Potts model in random fields has an interesting structure, and we hope that the exactness of the mean-field solution gives us help in understanding more realistic models.

In the next section the Potts model in random fields is defined. The model is solved by the mean-field method for two types of field distributions in Secs. III and IV. The explicit solution in one dimension for infinite $q$ is presented in Sec. V. It is shown that the mean-field approximation is exact in one dimension if $q$ is infinite. Section VI is devoted mainly to intuitive arguments on the effects of random fields similar to those for the Ising model.[8]

## II. POTTS MODEL IN RANDOM FIELDS

The $q$-state Potts model without randomness is described by the Hamiltonian

$$H=-J\sum_{\langle ij\rangle}\delta_{\lambda_i,\lambda_j}-h\sum_i\delta_{\lambda_i,1}\ , \tag{1}$$

where $\lambda_i$ takes on one of the $q$th roots of unity. By using the complex representation of Kronecker's $\delta$,

$$\delta_{\lambda_i,\lambda_j}=q^{-1}\sum_{r=0}^{q-1}\lambda_i^r\lambda_j^{q-r}\ , \tag{2}$$

we may write (1) as[9]

$$H=-q^{-1}J\sum_{\langle ij\rangle}\sum_{r=1}^{q-1}\lambda_i^r\lambda_j^{q-r}-q^{-1}h\sum_i\sum_{r=1}^{q-1}\lambda_i^r\ , \tag{3}$$

which differs from (1) by a trivial constant. A natural generalization of (3) to a random-field model is[10]

$$H=-q^{-1}J\sum_{\langle ij\rangle}\sum_r\lambda_i^r\lambda_j^{q-r}-q^{-1}\sum_i\sum_r\lambda_i^r h_i^{(q-r)}\ , \tag{4}$$

where $h_i^{(r)}$ are quenched random variables satisfying $h_i^{(r)}=(h_i^{(q-r)})^*$ so that (4) is real. We will be concerned with two types of distribution functions for $h_i^{(r)}$. The first one is the discrete distribution

$$h_i^{(r)}=h\tau_i^r\ , \tag{5}$$

where randomness is carried by $\tau_i$ which is quenched to one of the $q$th roots of unity at each site. Any one of the $q$th roots has equal probability $q^{-1}$ to be taken by $\tau_i$. In terms of Kronecker's $\delta$, the system (4) with the discrete distribution (5) is

$$H=-J\sum_{\langle ij\rangle}\delta_{\lambda_i,\lambda_j}-h\sum_i\delta_{\lambda_i,\tau_i}+\text{const}\ . \tag{6}$$

Equation (6) makes it clear that the discrete distribution (5) is a natural generalization of the binary distribution of fields for the random-field Ising model. The Gaussian distribution of $h_i^{(r)}$ has the probability function

$$P(\{h_i^{(r)}\}_r)=(2\pi\sigma^2)^{-(q-1)}\times\exp\left[-\sum_r h_i^{(r)}h_i^{(q-r)}/2\sigma^2\right]\ , \tag{7}$$

over which the free energy is averaged.

The effective Hamiltonian of the mean-field approximation to (4) is[3]

$$H_0=\tfrac{1}{2}q^{-1}(q-1)NJm^2c-q^{-1}mcJ\sum_i\sum_r\lambda_i^r-q^{-1}\sum_i\sum_r\lambda_i^r h_i^{(q-r)}\ , \tag{8}$$

where $N$ is the system size and $c$ denotes the coordination number. In the following sections we solve (8) for two types of field distributions, (5) and (7). Before solving, we

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should remark that the mean-field approximation is exact if the interaction range is infinite, because the difference of the original Hamiltonian (4) and the effective Hamiltonian (8) does not include the random-field term, and thus the conventional argument for the exactness of the mean-field approximation[11] applies.

## III. DISCRETE DISTRIBUTION

If the randomness is discrete (5), the mean-field Hamiltonian (8) becomes, with the aid of (2),

$$H_0=\tfrac{1}{2}q^{-1}(q-1)NJm^2c+q^{-1}mcJN+q^{-1}hN$$
$$-mcJ\sum_i \delta_{\lambda_i,1}-h\sum_i \delta_{\lambda_i,\tau_i}\ ,$$

from which the free energy averaged over randomness is found to be

$$\begin{aligned} f\equiv\beta F/N=&\tfrac{1}{2}(q-1)m^2cK+mcK+q^{-1}\beta h\\ &-q^{-1}\ln(e^{qmcK+\beta h}+q-1)\\ &-q^{-1}(q-1)\ln(e^{qmcK}+e^{\beta h}+q-2)\ , \end{aligned} \tag{9}$$

where $K=\beta J/q$. The parameter $m$ is determined by the minimization condition $\delta f/\delta m=0$ which is, explicitly, from (9),

$$m=\frac{e^{qmcK}}{e^{qmcK}+e^{\beta h}+q-2}-\frac{1}{e^{qmcK+\beta h}+q-1}\ . \tag{10}$$

The right-hand side of (10) can be verified to be the averaged value of $\lambda_i$, and therefore (10) represents the self-consistent relation. Equation (10) can be solved numerically without difficulty, and one finds the ferromagnetic ($m>0$) and paramagnetic ($m=0$) solutions depending on the values of parameters. Results are given in Fig. 1 for $q=3$ and 10 where the two phases are separated by a first-order transition line. In Fig. 1 the variables are scaled as

$$\bar K=qK/\ln q,\quad \bar h=h/J \tag{11}$$

for later convenience.

Of particular interest is the limit $q\to\infty$ since the mean-field approximation (8) may be exact even for finite-dimensional systems in this limit as indicated in Sec. V. When $q$ is very large, the free energy (9) reduces to

$$\bar f\equiv f/\ln q=\tfrac{1}{2}m^2c\bar K-\ln(q^{mc\bar K}+q^{\bar K\bar h}+q)/\ln q\ , \tag{12}$$

where the scaled parameters (11) have been used. Equation (12) is evaluated in the following manner. If $mc\bar K>\bar K\bar h$ and $mc\bar K>1$, (12) for $q\gg 1$ becomes

$$\bar f=\tfrac{1}{2}(m-1)^2c\bar K-\tfrac{1}{2}c\bar K\ , \tag{13}$$

which is minimum at $m=1$. The minimum value of $\bar f$ is

$$\bar f(m=1)=-\tfrac{1}{2}\bar K c\ . \tag{14}$$

To be consistent, the conditions $mc\bar K>\bar K\bar h$ and $mc\bar K>1$ must be satisfied at $m=1$:

$$\bar h<c,\quad \bar K^{-1}<c\ . \tag{15}$$

Only in the region (15) the solution $m=1$ of the self-consistent relation (10) exists. When $\bar K\bar h>mc\bar K$ and $\bar K\bar h>1$, the free energy is

$$\bar f=\tfrac{1}{2}m^2c\bar K-\bar K\bar h\ , \tag{16}$$

which has its minimum at $m=0$ with the minimum value

$$\bar f(m=0)=-\bar K\bar h\ . \tag{17}$$

The condition for (17) to be valid is

$$\bar K\bar h>1\ . \tag{18}$$

The third expression of the free energy takes place when $1>\bar K\bar h$ and $1>mc\bar K$. Then

$$\bar f=\tfrac{1}{2}m^2c\bar K-1 \tag{19}$$

with the minimum at $m=0$ and

$$\bar f(m=0)=-1\ . \tag{20}$$

This time the condition for the solution (20) to be valid is

$$1>\bar K\bar h\ . \tag{21}$$

The phase diagram can be drawn from these results (Fig. 2). First, if (15) is not satisfied, that is, if $\bar h>c$ or $\bar K^{-1}>c$, we have only the paramagnetic solutions (17) and (20). Equation (17) is valid in the region (18) while (20) should be taken if (21) is satisfied. These two values of the free energy match at $\bar K\bar h=1$ where the system undergoes a first-order transition. Although both states on the two sides of the transition at $\bar K\bar h=1$ are paramagnetic, they are qualitatively different from each other as is evident from the difference of the free energies (17) and (20). This point will be discussed later in more detail by introducing an order parameter to distinguish these two states.

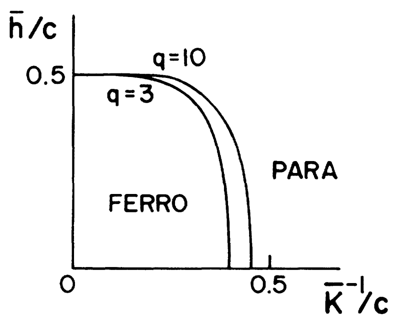


FIG. 1. Phase diagram in the case of the discrete field distribution (5) is obtained by numerically solving (10). The ferromagnetic and paramagnetic phases are separated by a first-order transition line.

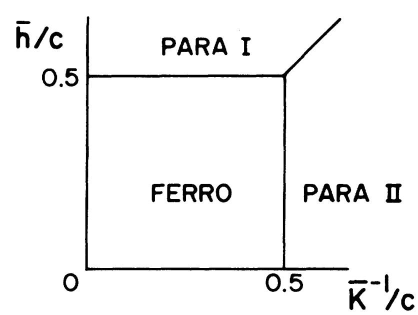


FIG. 2. Phase diagram of the infinite-state model with the discrete field distribution (5). The paramagnetic phase splits into two distinct states. All phases are separated from each other by first-order transitions.

If (15) is satisfied, we have to take account of the ferromagnetic solution (14). When (15) and (18) are satisfied at the same time, we compare (14) with (17) to obtain the transition point. The result is

$$\bar{h}=\tfrac{1}{2}c \;, \tag{22}$$

which satisfies the conditions $\bar{h}<c$ and $\bar{K}^{-1}<c$ when (18) is valid. Thus in the region where (15) and (18) are simultaneously valid, a first-order transition exists on the line (22) between the paramagnetic and ferromagnetic states. On the other hand, if (15) and (21) are satisfied, the transition point is obtained by comparing (14) and (20). The paramagnetic state changes to the ferromagnetic one on the line

$$\bar{K}^{-1}=\tfrac{1}{2}c \;. \tag{23}$$

Therefore the phase diagram has three states, the ferromagnetic with (14), the paramagnetic I with (17), and the paramagnetic II with (20) in respective regions (Fig. 2).

To distinguish the two paramagnetic states, it is convenient to introduce the following order parameter[10]:

$$Q=[\langle \lambda_i^r\rangle\langle \lambda_i^{q-r}\rangle] \;, \tag{24}$$

where the square brackets denote the configurational average. $Q$ is a measure of ordering along random fields. The order parameter $Q$ is a natural generalization of the "spin-glass" order parameter for the Ising model under random fields[3] although the spin-glass is not an appropriate expression to describe the present situation. Since we intend to distinguish the two paramagnetic states, we may assume $m=0$ to calculate (24) ($Q$ turns out to be 1 for $q\to\infty$ in the ferromagnetic phase). If $m=0$, an explicit evaluation of (24) yields

$$Q=(e^{\beta h}-1)^2/(e^{\beta h}+q-1)^2 \;. \tag{25}$$

With the scaling (11), it is easy to verify in the limit $q\to\infty$ that $Q=1$ if $\bar{K}\bar{h}>1$ while $Q=0$ if $\bar{K}\bar{h}<1$. Hence the paramagnetic phase I has the perfect ordering along random fields while the other phase II is thermally disturbed. It should be noted here that $Q$ is always finite if $\beta h$ and $q$ are both finite. It follows that the paramagnetic phase II characterized by $Q=0$ does not exist for finite $q$ unless $\beta h=0$.

## IV. GAUSSIAN DISTRIBUTION

If the random fields obey the Gaussian distribution (7), the free energy can be written as

$$f=\tfrac{1}{2}(q-1)m^2cK-(2\pi)^{-(q-1)}\int\prod_r dh^{(r)}\exp\left[-\frac{1}{2}\sum_r h^{(r)}h^{(q-r)}\right]\ln\mathrm{Tr}\exp\left[mcK\sum_r\lambda^r+\beta\sigma q^{-1}\sum_r\lambda^r h^{(q-r)}\right] \;, \tag{26}$$

where $K=\beta J/q$ and $h^{(r)}=(h^{(q-r)})^*$. The order parameter $m$ is determined from the condition $\delta f/\delta m=0$. Explicit numerical solution to this extremum condition yields the phase diagram for $q=3$ depicted in Fig. 3. We have to scale the variables as

$$qK=\bar{K}\ln q,\;\; \beta\sigma=\bar{K}\bar{\sigma}\sqrt{q\ln q} \tag{27}$$

to have a nontrivial phase diagram in the limit $q\to\infty$. The integral in (26) has precisely the same form as the integral evaluated in the Appendix of Ref. 10. Hence by a simple change of variables, $\bar{K}_0\to cK$ and $\bar{K}\sqrt{qQ}\to\beta\sigma q^{-1/2}$, from Ref. 10 to the present situation, we readily obtain the value of the free energy (26) in the limit $q\to\infty$. The result is as follows. First suppose $\bar{K}\bar{\sigma}>\sqrt{2}$. Then the free energy of the paramagnetic state is

$$\bar{f}(m=0)=-4\sqrt{2\pi}e^{-2}\bar{K}\bar{\sigma} \;, \tag{28}$$

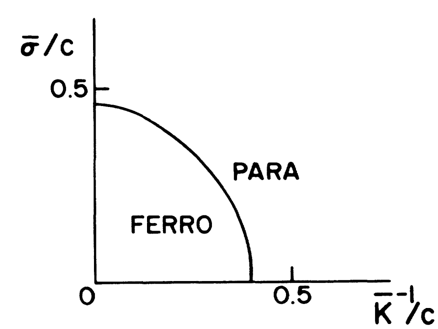


FIG. 3. Phase diagram for the Gaussian distribution (7) when $q=3$. A first-order boundary separates two phases.

and that of the ferromagnetic state is[12]

$$\bar{f}(m=1)=-\tfrac{1}{2}\bar{K}c \;. \tag{29}$$

A first-order transition line is therefore at

$$\bar{\sigma}=(8\sqrt{2\pi}e^{-2})^{-1}c \;. \tag{30}$$

In the region $\bar{K}\bar{\sigma}<1/\sqrt{2}$, $\bar{f}$ is found to be

$$\bar{f}=\tfrac{1}{2}m^2c\bar{K}-\tfrac{1}{2}\bar{K}^2\sigma^2-1 \tag{31}$$

if

$$A\equiv\bar{K}\bar{\sigma}/2+1/\bar{K}\bar{\sigma}-mc/\bar{\sigma}>0 \;. \tag{32}$$

The free energy (31) is minimum at $m=0$ and

$$\bar{f}(m=0)=-\tfrac{1}{2}\bar{K}^2\sigma^2-1 \;. \tag{33}$$

The condition $A>0$ is always satisfied if $m=0$. When $A<0$,

$$\bar{f}=\tfrac{1}{2}c\bar{K}(m-1)^2-\tfrac{1}{2}c\bar{K} \;, \tag{34}$$

which is minimum at $m=1$ with the minimum value

$$\bar{f}(m=1)=-\tfrac{1}{2}c\bar{K} \;. \tag{35}$$

The condition $A(m=1)<0$ should be satisfied for (35) to be valid. The transition point is obtained by comparing (33) and (35) to yield

$$\bar{\sigma}^2=c\bar{K}^{-1}-2\bar{K}^{-2} \tag{36}$$

as the phase boundary. On the boundary (36), $A(m=1)<0$ is satisfied. Thus we have obtained the phase boundaries (30) and (36) between the ferromagnetic

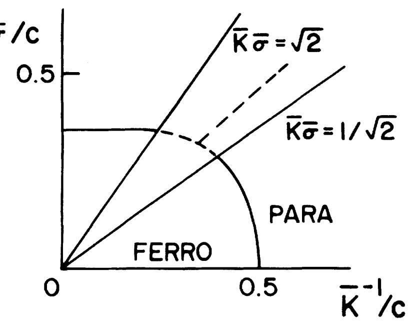


FIG. 4. Phase diagram of the infinite-state model with the Gaussian distribution (7). Although we cannot solve the problem in the region between $\bar{K}\bar{\sigma}=\sqrt{2}$ and $\bar{K}\bar{\sigma}=1/\sqrt{2}$, the phase diagram is expected to have a similar structure to Fig. 2.

and paramagnetic states in respective regions $\bar{K}\bar{\sigma}>\sqrt{2}$ and $\bar{K}\bar{\sigma}<1/\sqrt{2}$ (Fig. 4). We could not evaluate the free energy (26) for large $q$ in the intermediate region $1/\sqrt{2}<\bar{K}\bar{\sigma}<\sqrt{2}$. However, from the experience of the discrete case in the preceding section, we expect that a transition line exists in this intermediate region between the two paramagnetic phases.

## V. ONE-DIMENSIONAL SYSTEM

In this section we solve the random-field Potts model with discrete field distribution (5) in one dimension for large $q$. The result agrees with that of the mean-field approximation described in Sec. III. In the limit of infinite $q$, only a special class of random-field configurations gives a finite contribution to the averaged free energy. That is, no $\tau_i$ is equal to any other $\tau_j$ if the free energy for a given field configuration is to have a finite probability weight when we average over randomness. The reason is the following. As shown explicitly later, any field configuration satisfying the above condition has the same value of the free energy, irrespective of the precise configuration of $\{\tau_i\}$. Thus the total probability weight of the above class of field configurations is $q^{-N}q(q-1)(q-2)\cdots(q-N+1)$ which tends to 1 in the limit $q\to\infty$ with $N$ finite. Since this class of field configurations occupies the whole probability 1, no other configurations give a finite contribution.

Next we evaluate the free energy in one dimension for the above field configuration. The partition function of the present problem (6) can be written as

$$Z=\mathrm{Tr}\prod_i[1+(e^{\beta J}-1)\delta_{\lambda_i,\lambda_{i+1}}]\times\prod_i[1+(e^{\beta h}-1)\delta_{\lambda_i,\tau_i}] \ . \tag{37}$$

We ignored the constant term in (6) because it gives only a vanishing contribution in the limit $q\to\infty$ if we hold $\bar{K}$ and $\bar{h}$ in (11) fixed. As is well known,[13] if $h=0$, $Z$ is expressed as a sum over graphs $G$, each of which consists of several connected clusters. Within a cluster all $\lambda_i$ assume the same value because of Kronecker's $\delta$. After the trace over $\lambda_i$ is taken[13] (note $h=0$),

$$Z=\sum_G(e^{\beta J}-1)^b q^n \ , \tag{38}$$

where the sum is over all graphs $G$ on the one-dimensional chain, and $b$ and $n$ represent the number of bonds and clusters, respectively, in the graph. In one dimension we have $n+b=N$,[13] and therefore

$$Z=q^N\sum_G(e^{\beta J}-1)^b q^{-b} \ . \tag{39}$$

We next add the random-field contribution. In the expansion of (37) in terms of graphs, we cannot assign two field terms $(e^{\beta h}-1)\delta_{\lambda_i,\tau_i}$ to a single connected cluster because a $\tau_i$ is different from any other $\tau_j$ while $\lambda_i$ should be equal to each other in a cluster. Thus a cluster can have, at most, one contribution from the field term $(e^{\beta h}-1)\delta_{\lambda_i,\tau_i}$. The assigning of a field term to a cluster reduces a degree of freedom since $\lambda_i$ is locked to $\tau_i$. Therefore the field term always enters as a power of $(e^{\beta h}-1)/q$ after the trace over $\lambda_i$ is taken:

$$Z=q^N\sum_G(e^{\beta J}-1)^b q^{-b}(e^{\beta h}-1)^m q^{-m} \ , \tag{40}$$

where $m$ is the number of clusters with field contributions (recall that a cluster cannot have more than one field contribution on it). We now scale the parameters as in (11) and take the limit $q\to\infty$. Then

$$(e^{\beta J}-1)/q\approx q^{\bar{K}-1}$$

and (41)

$$(e^{\beta h}-1)/q\approx q^{\bar{h}\bar{K}-1} \ .$$

The leading term in the graphical expansion (40) is determined by the values of parameters. When $\bar{K}<1$ and $\bar{h}\bar{K}<1$, both quantities in (41) vanish in the $q\to\infty$ limit. Thus the graph with finite contribution in (40) has $b=m=0$, that is, $Z=q^N$. The (reduced) free energy for this $Z$ is

$$\bar{f}=-1 \ . \tag{42}$$

Apparently the graph with $b=m=0$ represents the completely thermally disturbed state because this graph can also be obtained from the condition $\beta=0$. The free energy (42) is that of the paramagnetic phase which occupies the region $\bar{K}<1$ and $\bar{h}\bar{K}<1$ (see Fig. 2). If $\bar{K}>1$ and $\bar{h}\bar{K}<1$, $q^{\bar{K}-1}\gg 1$ but $q^{\bar{h}\bar{K}}\ll 1$. Hence the leading graph has $b=N-1$ and $m=0$ where the free boundary condition was assumed. The partition function and the free energy are

$$Z=q^N q^{(\bar{K}-1)(N-1)}$$

and

$$\bar{f}=-(N-1)\bar{K}/N-1/N\approx-\bar{K} \ , \tag{43}$$

where $N$ is assumed to be large (note that the limit $N\to\infty$ was taken *after* $q\to\infty$). The graph with $b=N-1$ and $m=0$ represents the ferromagnetic state. The third region of the phase diagram is defined by the inequalities $\bar{h}\bar{K}>1$ and $\bar{h}\bar{K}>\bar{K}$. In this region $q^{\bar{h}\bar{K}-1}\gg 1$ and $q^{\bar{h}\bar{K}-1}\gg q^{\bar{K}-1}$. Thus we have $b=0$ and $m=N$ to yield

$$\bar{f}=-\bar{h}\bar{K} \ , \tag{44}$$

which represents the paramagnetic state disturbed by the random fields (paramagnetic I in Fig. 2). In the last region $\bar{h}\bar{K}>1$ and $\bar{h}\bar{K}<\bar{K}$, the leading graph has $b=N-1$ and $m=1$, and the free energy is

$$\bar{f} = -(N-1)\bar{K}/N - \bar{h}\bar{K}/N \approx -\bar{K} \; . \tag{45}$$

Therefore the free energies (42)–(45) agree with those obtained by the mean-field theory (with $c=2$) in respective regions.

## VI. DISCUSSION

The Potts model was generalized to include random fields. It was solved by the mean-field approximation. In the limit of large $q$ we could solve the model explicitly in one dimension to find that the mean-field approximation is exact. Since the mean-field approximation is, in general, exact also in the limit of large spatial dimensionality, we may expect that this approximation is exact for any $d$ if $q$ is infinite, as is the case for the nonrandom Potts model.[7]

Another interesting fact about the infinite-state Potts model in random fields is that the ferromagnetic phase is stable against small (but finite) random fields even in one dimension, which is not the case in other models investigated so far.[1–5] A simple energetics argument, similar to that of Imry and Ma,[8] can explain this stability intuitively. Suppose that the system is split into domains of typical linear size $L$ in the ground state. The boundary energy of a domain is about $JL^{d-1}$. Although the field term in our Hamiltonian (4) vanishes on average, it has finite fluctuations. If the distribution is discrete (5), the typical scale of fluctuations is found to be $q^{-1/2}hL^{d/2}$. Consequently, if we fix $J$ and $h$ and take the limit $q \to \infty$ as we did in Sec. III, the contribution of the field terms vanishes, which implies the stability of the ordered phase. It should be noted that the limit $L \to \infty$ is taken, if necessary, *after* $q \to \infty$. For the Gaussian distribution (7), the order of field fluctuations is $\bar{\sigma}L^{d/2}(\ln q)^{-1/2}$, which also vanishes in the limit $q \to \infty$ if we hold $\bar{\sigma}$ fixed [see (27) for the definition of $\bar{\sigma}$]. In the case of the Ising model it is argued[4] that the roughness of domain interfaces reduces the stability of the ordered phase. That is, on the boundary between two domains the Ising spins adjust themselves so that the maximal random-field (negative) energy gain is realized. This phenomenon apparently enhances the field effects, thus destabilizing the ordered phase. As for the infinite-state Potts model, it does not seem that this sort of interface roughness works to destroy the ferromagnetic state. Since the probability of realizing a particular value of $\tau_i$ is vanishingly small, proportional to $q^{-1}$, it is impossible for a boundary spin to find itself in the direction parallel to the applied field. Therefore the destabilizing mechanism of interface roughness does not work in the infinite-state Potts model, which justifies the validity of the simple energetics explained above. Although our main interest in this paper was in the infinite-state limit $q \to \infty$, it may be possible to investigate the case of finite $q$ by the method of $q^{-1}$ expansion[14] which has turned out to be extremely useful for nonrandom models.

After this paper was submitted for publication, the author was informed that Blankschtein *et al.*[15] recently discussed the same subject. The results of the present paper agree with theirs where overlap is found. They carried out mean-field analysis for finite $q$, similar to ours, to find first-order transitions. They further discussed the shift of critical dimension. However, they gave no argument about the infinite-state limit, which is one of the major points of this paper.

## ACKNOWLEDGMENTS

Discussions with E. Abrahams and M. Chang are gratefully acknowledged. I would like to also thank M. J. Stephen for discussions on related topics and Y. Shapir for calling my attention to Ref. 15.


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