Journal of the Physical Society of Japan
Vol. 58, No. 9, September, 1989, pp. 3433-3434

More about the Ground State of Quantum Spin Systems on the Triangular Lattice

Hidetoshi NISHIMORI and Hiizu NAKANISHI†

Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152
†Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, U.S.A.

(Received June 2, 1989)

We report here some additional analysis on the ground state properties of the quantum XY and Heisenberg models on the triangular lattice using the data of finite-size lattices $N \leq 27$.[1] We refer the reader to our original paper[1] for notation and detailed definitions of various quantities.

It was proved recently[2] that the XY model on the square lattice has finite long range order. This rigorous result apparently conflicts with our prediction of the absence of long range order in the ferromagnetic XY model on the triangular lattice;[1] it is highly unlikely, when the interaction is ferromagnetic, that the triangular-lattice ferromagnet fails to have long range order while the square-lattice counterpart does have finite ordering. For this reason we have tried an additional fitting function suggested from the spin-wave theory[3] to explain the finite-size data of the order parameter

$$f_3=0.81719+0.85108/\sqrt{N}-0.97219/N. \quad (1)$$

The values of the order parameter calculated from (1) for finite-size systems are listed in Table I as well as those from the other fitting functions given in the original paper,

$$f_1=0.90383+1.30960/N-4.49411/N^2,$$
$$f_2=N^{-0.11906/2}(1.15896-0.25321/N).$$

It is clear from Table I that f_1 is less appropriate than the others, but we can hardly judge which of f_2 and f_3 is better. Therefore it is dangerous to draw a definite conclusion on the infinite-system long range order only from the data of finite-size systems as small as $N \leq 27$. This fact has already been stressed in §5 of the original paper. In any case, the value of the order parameter of the infinite system will not be very far from 0.817 (the leading term in (1)) if the spin wave picture is valid.

Table I. Long-range order in the ferromagnetic XY model and the residuals (fitting errors).

N	data($\equiv f$)	$(f-f_1)\times 10^5$	$(f-f_2)\times 10^5$	$(f-f_3)\times 10^5$
7	1	81	3	1
9	0.99199	−186	−19	−87
12	0.98162	−13	23	−24
13	0.97835	38	23	−11
16	0.96893	81	−27	−27
19	0.96145	115	2	18
21	0.95666	66	−12	4
25	0.94837	−65	−13	−15
27	0.94500	−116	22	2

The finite-size data of the twisted magnetization for the antiferromagnetic XY model (Table II) have also been analyzed by the spin wave picture,

$$n^2=0.26310+1.1914/\sqrt{N}-1.2871/N,$$

which has been designed to recover correctly the values for $N=9$, 21 and 27. The only plausible statement to be made here is that the infinite-system long-range order would be close to 0.263 if the spin wave theory is correct.

The original analysis of the twisted magnetization of the antiferromagnetic Heisenberg model (Table II) contained an error (see ref. 4). Equations (4.8) to (4.11) of the original paper should be modified as follows:

$$n^2=0.099179+8.4567/N-36.144/N^2 \quad (4.8)$$
$$n^2=N^{-0.74280}(4.7795-15.738/N) \quad (4.9)$$
$$n^2=-14.110/N-135.22/N^2+506.109/N^3 \quad (4.10)$$
$$n^2=-0.81850/N+1.4524N^{0.0027693} \times \exp(-0.25326\sqrt{N}). \quad (4.11)$$

These equations, in particular (4.8), suggest a larger possibility of finite ordering as com-

Table II. Twisted magnetization n^2 of antiferromagnets.

N	XY model	Heisenberg model
3	0.55556	0.66667
9	0.51723	0.59259
21	0.46180	0.41992
27	0.44472	0.36281

pared to the corresponding equations appeared in the original paper. However, our present standpoint is that the system size $N=27$ or less is too small to draw reliable conclusions by simple extrapolation as given above. We have also fitted the data of $N=9$, 21 and 27 to the spin wave form,

$$n^2=-0.27726+4.3045/\sqrt{N}-5.0847/N.$$

It is not a simple matter to judge whether the negative value of the leading term (which should correspond to the infinite-lattice value of the order parameter) is an artifact of smallness of the lattice size or an suggestion for the essential inappropriateness of this function (the latter possibility implies the breakdown of the spin wave theory).

One of the authors (H.N.) thanks A. P. Young for correspondence on the finite-size data of the antiferromagnetic Heisenberg model.

References

1) H. Nishimori and H. Nakanishi: J. Phys. Soc. Jpn. **57** (1988) 626.
2) K. Kubo and T. Kishi: Phys. Rev. Lett. **61** (1988) 2585. T. Kennedy, E. H. Lieb and B. S. Shastry: Phys. Rev. Lett. **61** (1988) 2582.
3) R. D. Reger and A. P. Young: Phys. Rev. **B37** (1988) 5978.
4) H. Nishimori and H. Nakanishi: J. Phys. Soc. Jpn. **58** (1989) 2607. (Erratum of ref. 1)