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Absence of Ordering in Short Range Vector Spin Glass: Revisited

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We show the absence of spin glass ordering in short range vector spin systems with random symmetric exchange interactions in spatial dimension $d \leqq 4$. The argument is an improvement of our previous theory; we circumvent the use of the unproved assumption which has been shown not to hold by O'Neill and Moore. Still, we have not completed a mathematically rigorous proof because of the use of the replica method and of the unproved clustering property of spin correlation functions.

§1. Introduction

Most of the spin glass physicists now believe that the short range XY and Heisenberg models do not undergo a spin glass transition at a finite temperature in three dimensions.[1-8) It is highly desirable to prove this belief by a mathematically rigorous method. In a previous paper[9] (referred to as ON hereafter), the present authors pointed out a difficulty in Schuster's argument,[10] which is basically the Mermin-type[11] argument using the Bogoliubov inequality, on the absence of spin glass ordering in the XY and Heisenberg models for spatial dimension $d \leqq 4$. We showed how to improve his argument by appropriately modifying definitions of various quantities. ON further showed that the system with power-decaying interactions (such as the RKKY interaction) can be treated in the same framework, and reproduced a phase diagram proposed from renormalization group calculations.[6] ON's argument was not rigorous in, first, that the replica method was used, and second, that an unconfirmed assumption was used on the parameter dependence of a term appearing in the inequality. We supposed that this second assumption was reasonable since the final results on the parameter region where the spin glass ordering does not appear were in quite good agreement with those derived from other methods. O'Neill and Moore[12] (OM) discussed, however, that ON's assumption is not satisfied in a short range system.

We show in the present paper that the difficulty pointed out by OM can be avoided by slightly changing definition of the terms appearing in the inequality. The argument is restricted to the short range model.

§2. Definition

Most of the definition and notation parallel those in ON. However, for self-containedness, the list of definition and notation is given here. We treat a system composed of classical vector spins on a d-dimensional lattice. The Hamiltonian is given by

$$\mathscr{H}=\mathscr{H}_{\rm ex}+\mathscr{H}_{\rm f}+\mathscr{H}_{\rm D}, \tag{2.1}$$

$$\mathscr{H}_{\rm ex}=-\sum_{\langle ij\rangle} J_{ij}(S_{ix}S_{jx}+S_{iy}S_{jy}+\varDelta S_{iz}S_{jz}), \tag{2.2}$$

$$\mathscr{H}_{\rm f}=-h\sum_{i}\xi_i S_{ix}, \tag{2.3}$$

$$\mathscr{H}_{\rm D}=-D\sum_{i}(S_{iz})^2. \tag{2.4}$$

The range of interaction is short, which is symbolized by the bracketed pair $\langle ij\rangle$. The coupling J_{ij} is a quenched random variable with the symmetric Gaussian distribution function of variance J^2. $\mathscr{H}_{\rm f}$ represents a symmetry breaking random field along the x-axis. The distribution of ξ_i is assumed to be symmetric Gaussian with variance unity. $\mathscr{H}_{\rm D}$ represents uniaxial anisotropy.

It is convenient to introduce canonical variables L_i and θ_i defined by

$$S_{ix}=(S^2-L_i^2)^{1/2}\cos\theta_i, \tag{2.5a}$$

$$S_{iy}=(S^2-L_i^2)^{1/2}\sin\theta_i, \tag{2.5b}$$

$$S_{iz}=L_i. \tag{2.5c}$$

Using the replica method,[13-15] we derive the effective Hamiltonian $\mathscr{H}'(n)$ defined by

$$\mathrm{Tr}\exp(-\beta\mathscr{H}'(n))\equiv[Z(\{J_{ij}\},\{\xi_i\})^n]_c, \tag{2.6}$$

where $\beta=1/k_BT$. The square brackets $[\ ]_c$ denote the configurational average over the distribution of $\{J_{ij}\}$ and $\{\xi_i\}$, and Tr is for the integration over the canonical variables $\{\theta_i^\alpha, L_i^\alpha\}$, where $\alpha(=1,\cdots,n)$ is the replica index. After performing the configurational average, one finds

$$\mathscr{H}'(n)=\mathscr{H}'_{ex}(n)+\mathscr{H}'_f(n)+\mathscr{H}'_D(n), \tag{2.7}$$

$$\mathscr{H}'_{ex}(n)=-\frac{\beta}{2}J^2\sum_{\langle ij\rangle}\times\{\sum_\alpha(S_{ix}^\alpha S_{jx}^\alpha+S_{iy}^\alpha S_{jy}^\alpha+\Delta S_{iz}^\alpha S_{jz}^\alpha)\}^2, \tag{2.8}$$

$$\mathscr{H}'_f(n)=-\frac{\beta}{2}h^2\sum_i(\sum_\alpha S_{ix}^\alpha)^2, \tag{2.9}$$

$$\mathscr{H}'_D(n)=-D\sum_i\sum_\alpha(S_{iz}^\alpha)^2. \tag{2.10}$$

The prime on $\mathscr{H}$, $\mathscr{H}_{ex}$ etc. will be omitted hereafter to simplify the notation. The spin glass order parameter q can be calculated as

$$q=\lim_{h\to0}\lim_{N\to\infty}\lim_{n\to0}q(h,N,n), \tag{2.11}$$

where N is the system size and

$$q(h,N,n)=\langle S_{ix}^\gamma S_{ix}^\delta\rangle.\quad(\gamma\neq\delta) \tag{2.12}$$

Here the angular brackets represent the average

$$\langle\cdots\rangle=\mathrm{Tr}\exp(-\beta\mathscr{H}(n))\cdots. \tag{2.13}$$

§3. Absence of Ordering

For simplicity of notation, we restrict ourselves to the XY model ($S=1$, $\Delta=0$, and $L_i=0$ at all i) in the present section. No essential change is necessary when one generalizes the argument to an arbitrary model expressed by the Hamiltonian (2.1)-(2.4): One should only multiply appropriate expressions in the following arguments by products of $(S^2-L_i^2)^{1/2}$.

We make use of the Schwartz inequality

$$\langle AA^*\rangle\geqq|\langle A[C^*,\mathscr{H}]\rangle|^2/\langle|[C,\mathscr{H}]|^2\rangle, \tag{3.1}$$

where A and C are periodic functions of the variables $\{\theta_i^\alpha\}$, and the Poisson bracket is defined by

$$[X,Y]=\frac{1}{2}\sum_{ij}\sum_{\alpha\beta}\left(\frac{\partial^2X}{\partial\theta_i^\alpha\partial\theta_j^\beta}\frac{\partial^2Y}{\partial L_i^\alpha\partial L_j^\beta}-\frac{\partial^2Y}{\partial\theta_i^\alpha\partial\theta_j^\beta}\frac{\partial^2X}{\partial L_i^\alpha\partial L_j^\beta}\right). \tag{3.2}$$

Following Schuster,[10] we choose A and C as

$$A=\sum_j\exp(-i\boldsymbol{k}\cdot\boldsymbol{R}_j)\sin\theta_j^\gamma\sin\theta_j^\delta,\quad(\gamma\neq\delta) \tag{3.3}$$

$$C=\sum_{lm}\exp\{-i\boldsymbol{k}\cdot(\boldsymbol{R}_l+\boldsymbol{R}_m)/2\}L_l^\gamma L_m^\delta.\quad(\gamma\neq\delta) \tag{3.4}$$

We first note that the anisotropy $\mathscr{H}_D$ does not enter the calculations of the inequality (3.1) because the Poisson bracket of C and $\mathscr{H}_D$ identically vanishes from the definition of these two quantities.

Let us evaluate the denominator on the right hand side (rhs) of (3.1). In the limit of small wave number $k\to0$ and small field $h\to0$, the denominator is found to behave as

$$\langle|[C,\mathscr{H}]|^2\rangle=c_1k^4N+c_2k^2h^2N+c_3h^4N, \tag{3.5}$$

where c_1, c_2 and c_3 are functions of β (independent of k and N). c_1 and c_3 are positive while we are not aware of the sign of c_2. The reason is as follows.

The exchange term contribution is

$$\langle|[C,\mathscr{H}_{ex}]|^2\rangle=\sum_{lm}\sum_{rs}\exp\{-i\boldsymbol{k}\cdot(\boldsymbol{R}_l+\boldsymbol{R}_m-\boldsymbol{R}_r-\boldsymbol{R}_s)/2\}\left\langle\frac{\partial^2\mathscr{H}_{ex}}{\partial\theta_l^\gamma\partial\theta_m^\delta}\frac{\partial^2\mathscr{H}_{ex}}{\partial\theta_r^\gamma\partial\theta_s^\delta}\right\rangle. \tag{3.6}$$

This is exactly the second term on the rhs of (2.13) of OM. Thus their analysis of dependence on the wave number and the system size applies. It is useful to repeat their consideration here to clarify the limit of applicability of their argument. By carrying out the differentiation, we find that (3.6) in the limit of small k reduces to

$$\beta^2 J^4 \sum_{\langle ij\rangle}\sum_{\langle bc\rangle}\langle\sin(\theta_i^\gamma-\theta_j^\gamma)\sin(\theta_b^\gamma-\theta_c^\gamma)\sin(\theta_i^\delta-\theta_j^\delta)\sin(\theta_b^\delta-\theta_c^\delta)\rangle[\boldsymbol{k}\cdot\boldsymbol{R}_{ij}/2]^2[\boldsymbol{k}\cdot\boldsymbol{R}_{bc}/2]^2. \tag{3.7}$$

A first glance at (3.7) reveals superficial dependence on the system size as N^2 because of the double summation over $\langle ij\rangle$ and $\langle bc\rangle$. To see the size dependence more precisely, let us separate the pairs $\langle ij\rangle$ and $\langle bc\rangle$ far apart from each other. Then, if the clustering property of the spin correlation function holds, the correlation function in (3.7) in the well-separated limit is

$$\langle\sin(\theta_i^\gamma-\theta_j^\gamma)\sin(\theta_i^\delta-\theta_j^\delta)\rangle\langle\sin(\theta_b^\gamma-\theta_c^\gamma)\sin(\theta_b^\delta-\theta_c^\delta)\rangle,$$

which vanishes because of the global inversion symmetry $\theta_i^\delta\to-\theta_i^\delta$ at all i in the first factor. Therefore the double summation in (3.7) yields finite contribution only when the pairs $\langle ij\rangle$ and $\langle bc\rangle$ are close to each other. This implies that (3.7) is proportional to N, and hence we have derived the first term on the rhs of (3.5).

The above argument breaks down if the spin correlation function in (3.7) does not cluster in a sufficiently large system. The spin correlation function defined by a simple statistical mechanical average may not cluster if the low temperature phase (if any) has a multivalley structure:[16] An average within a valley (or a Gibbs state) clusters[17] whereas an average over the whole phase space does not. We thus have to make an explicit statement here that the existence of an ordered phase with a multivalley structure is not excluded on the basis of the present argument. Another remark is made on the N-dependence of the rhs of (3.7). If the correlation function decays in a power law as $\langle ij\rangle$ and $\langle bc\rangle$ are separated, (3.7) may have stronger dependence on N than simple linear-N behavior even when clustering holds. Hence we assume here that the connected correlation function

$$\langle\sin(\theta_i^\gamma-\theta_j^\gamma)\sin(\theta_b^\gamma-\theta_c^\gamma)\sin(\theta_i^\delta-\theta_j^\delta)\sin(\theta_b^\delta-\theta_c^\delta)\rangle$$
$$-\langle\sin(\theta_i^\gamma-\theta_j^\gamma)\sin(\theta_i^\delta-\theta_j^\delta)\rangle\langle\sin(\theta_b^\gamma-\theta_c^\gamma)\sin(\theta_b^\delta-\theta_c^\delta)\rangle,$$

decays exponentially as the pairs $\langle ij\rangle$ and $\langle bc\rangle$ are separated.

The field term contribution to the Poisson bracket in (3.5) is

$$[C,\mathscr{H}_{\mathrm{f}}]=\beta h^2\sum_j\exp(-i\boldsymbol{k}\cdot\boldsymbol{R}_j)\sin\theta_j^\gamma\sin\theta_j^\delta.$$

Thus, together with the above result on $[C,\mathscr{H}_{\mathrm{ex}}]$, we have

$$\begin{aligned}\langle|[C,\mathscr{H}_{\mathrm{ex}}+\mathscr{H}_{\mathrm{f}}]|^2\rangle=&c_1k^4N+2\beta^2J^2h^2\sum_l\exp(-i\boldsymbol{k}\cdot\boldsymbol{R}_l)\sum_{\langle ab\rangle}\{\exp(i\boldsymbol{k}\cdot\boldsymbol{R}_a/2)\\&-\exp(i\boldsymbol{k}\cdot\boldsymbol{R}_b/2)\}^2\langle\sin\theta_l^\gamma\sin\theta_l^\delta\sin(\theta_a^\gamma-\theta_b^\gamma)\sin(\theta_a^\delta-\theta_b^\delta)\rangle\\&+\beta^2h^4\sum_{jl}\exp\{-i\boldsymbol{k}\cdot(\boldsymbol{R}_j-\boldsymbol{R}_l)\}\langle\sin\theta_j^\gamma\sin\theta_j^\delta\sin\theta_l^\gamma\sin\theta_l^\delta\rangle.\end{aligned}\tag{3.8}$$

From the same argument as above (i.e., using the clustering of spin correlation functions), we find that all terms on the rhs of (3.8) is of $O(N)$. The dependence of the second and third terms on the rhs of (3.8) on k and h are easily read out as k^2h^2 and h^4 in the leading order. This completes the derivation of (3.5). Since the first and the third terms on the rhs of (3.8) are both squares of certain quantities, the coefficients c_1 and c_3 in (3.5) are positive.

Next, we evaluate the numerator on the rhs of (3.1). The explicit form is

$$\langle A[C^*,\mathscr{H}]\rangle=-\sum_{lm}\exp\{i\boldsymbol{k}\cdot(\boldsymbol{R}_l+\boldsymbol{R}_m)/2\}\left\langle A\frac{\partial^2\mathscr{H}}{\partial\theta_l^\gamma\partial\theta_m^\delta}\right\rangle. \tag{3.9}$$

Evaluation of the expectation value on the rhs of (3.9) is carried out by integration by parts as follows (remember that the brackets $\langle\ \rangle$ denote the weighted integration over $\{\theta_i^\alpha, L_i^\alpha\}$, and only the former variables $\{\theta_i^\alpha\}$ play a role here):

$$\left\langle A\frac{\partial^2\mathscr{H}}{\partial\theta_l^\gamma\partial\theta_m^\delta}\right\rangle=-\left\langle\frac{\partial A}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}}{\partial\theta_m^\delta}\right\rangle+\beta\left\langle A\frac{\partial\mathscr{H}}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}}{\partial\theta_m^\delta}\right\rangle$$
$$=-\beta^{-1}\left\langle\frac{\partial^2 A}{\partial\theta_l^\gamma\partial\theta_m^\delta}\right\rangle+\beta\left\langle A\frac{\partial\mathscr{H}}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}}{\partial\theta_m^\delta}\right\rangle$$
$$=-\beta^{-1}\sum_j\delta_{lj}\delta_{mj}\exp(-i\boldsymbol{k}\cdot\boldsymbol{R}_j)\langle\cos\theta_l^\gamma\cos\theta_m^\delta\rangle+\beta\left\langle A\frac{\partial\mathscr{H}}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}}{\partial\theta_m^\delta}\right\rangle. \tag{3.10}$$

Accordingly, (3.9) becomes

$$\langle A[C^*,\mathscr{H}]\rangle=\beta^{-1}Nq(h,N,n)-\beta\sum_{lm}\exp\{i\boldsymbol{k}\cdot(\boldsymbol{R}_l+\boldsymbol{R}_m)/2\}\left\langle A\frac{\partial\mathscr{H}}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}}{\partial\theta_m^\delta}\right\rangle. \tag{3.11}$$

Since we will later investigate the infrared (small-k) divergence of the integration of the rhs of the Schwartz inequality (3.1), it is important to check the leading contribution as k approaches 0 in the second term on the rhs of (3.9). Let us thus consider the following quantity

$$\sum_{lm}\left\langle A\frac{\partial(\mathscr{H}_{\text{ex}}+\mathscr{H}_{\text{f}})}{\partial\theta_l^\gamma}\frac{\partial(\mathscr{H}_{\text{ex}}+\mathscr{H}_{\text{f}})}{\partial\theta_m^\delta}\right\rangle. \tag{3.12}$$

By explicit differentiation, we obtain

$$\frac{\partial\mathscr{H}_{\text{ex}}}{\partial\theta_l^\gamma}=-\beta J^2\sum_{\langle ij\rangle}\sum_\alpha(\delta_{li}-\delta_{lj})\cos(\theta_i^\alpha-\theta_j^\alpha)\sin(\theta_i^\gamma-\theta_j^\gamma),$$

the summation of which over l vanishes. Hence the corresponding terms in (3.12) can be neglected. The nonvanishing contribution comes from the field term in (3.12),

$$\sum_{lm}\left\langle A\frac{\partial\mathscr{H}_{\text{f}}}{\partial\theta_l^\gamma}\frac{\partial\mathscr{H}_{\text{f}}}{\partial\theta_m^\delta}\right\rangle=\beta^2h^4\sum_{lmj}\sum_{\alpha\beta}\langle\sin\theta_j^\gamma\sin\theta_j^\delta\sin\theta_l^\gamma\cos\theta_l^\alpha\sin\theta_m^\delta\cos\theta_m^\beta\rangle. \tag{3.13}$$

Equation (3.13) is proportional to h^4N according to the clustering property; if any of l, m, or j is separated far away from the others, the clustering property yields a vanishing factor. This fact implies that l, m and j must be close to each other to give nonvanishing contribution to (3.13). Therefore the rhs of (3.11) behaves as

$$(\beta^{-1}q+c_4h^4)N, \tag{3.14}$$

as $k\to 0$.

The final piece to be evaluated in the Schwartz inequality (3.1) is its lhs. It is straightforward to see

$$\frac{1}{N}\sum_k\langle AA^*\rangle\leqq N. \tag{3.15}$$

Substituting (3.5) and (3.14) into (3.1) and summing up over all k in the first Brillouin zone, with (3.15) taken into account, we obtain

$$1\geqq(q+c_4h^4)^2\int\frac{\mathrm{d}\boldsymbol{k}}{k^4+c_2h^2k^2+h^4}, \tag{3.16}$$

where irrelevant numerical factors have been dropped. The constants c_2 and c_4 have been kept in (3.16) because their signs are not known and so may not be reduced to 1 by appropriate rescaling.

It is not difficult to show that the integral (3.16) diverges for $d\leqq 4$ in the limit of small field.

There exists a unique value k_0 satisfying

$$k_0^4 = c_2 h^2 k_0^2 + h^4, \tag{3.17}$$

which can be verified by considering the k dependence of k^4 and $c_2h^2k^2+h^4$. Since $k^4 > c_2h^2k^2+h^4$ for $k > k_0$, we obtain

$$\int_0^a \frac{k^{d-1}\,\mathrm{d}k}{k^4+c_2h^2k^2+h^4} \geqq \int_{k_0}^a \frac{k^{d-1}\mathrm{d}k}{k^4+c_2h^2k^2+h^4} \geqq \int_{k_0}^a \frac{k^{d-1}\,\mathrm{d}k}{2k^4}.$$

Since k_0 approaches 0 as $h \to 0$ according to (3.17), the integral diverges as $h \to 0$ if $d \leqq 4$. This implies $q \to 0$ as $h \to 0$ if $d \leqq 4$ due to (3.16).

§4. Conclusion and Discussion

It has been shown that the spin glass order parameter q defined in (2.11) and (2.12) vanishes if $d \leqq 4$ for the short range vector spin system (2.1)-(2.4) in the limit of $h \to 0$ irrespective of the value of the uniaxial anisotropy D. The present argument circumvents the difficulty in ON pointed out by OM. There are still several points to be resolved before we reach the final proof. First, the use of the replica method remains to be justified. Second, the clustering property of spin correlation functions has not been established rigorously. If the phase space of the system possesses a multiple valley structure, as in the mean field model,[16] the correlation functions defined by the standard statistical mechanical average do not cluster. Thus our argument leads to the absence of ordering with a single valley structure (including the simple overall inversion symmetry), but a more complicated type of ordering could exist. It should also be added that the exponential decay of the connected correlation function is assumed in the order estimate of the sum of correlation functions.

If the distribution of J_{ij} is not Gaussian, higher order cummulants appear in addition to the second order cummulant (2.8) in $\mathscr{H}_{\mathrm{ex}}(n)$. We can develop the same argument as in §3 on the order estimate of the higher order cummulants. The conclusion that $q=0$ for $d \leqq 4$ remains unchanged.

It is not easy to treat the long range (power-decaying) interactions within the present framework. The main source of difficulty is in the evaluation of the N-dependence of the sum of spin correlation functions. Nevertheless, we believe that a minor improvement of the original ON argument, if necessary, would suffice to establish a satisfactory theory in view of the successful reproduction of the renormalization group phase boundary by ON.

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