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Boundary between the Ferromagnetic and Spin Glass Phases

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It is proved that the critical concentration p_c of ferromagnetic bonds below which no ferromagnetic phase exists in a non-Ising $\pm J$ model is equal to or larger than that of the Ising $\pm J$ model on the same lattice. I also develop an argument that p_c is universal for the $\pm J$ model with any type of spin on any specified lattice.

The $\pm J$ model of spin glasses has several phases. I argued in a previous paper[1] that the boundary separating the ferromagnetic phase and the non-ferromagnetic phase is parallel to the temperature axis. This conjecture has been supported for the Ising system by various approximate calculations including the renormalization group arguments,[2] numerical calculations,[3,4] series expansions,[5] and the coherent anomaly method,[6] as well as by experiments.[7] In view of the recent efforts to determine the precise phase diagram of non-Ising $\pm J$ models,[8] it may be useful to provide a proof that the critical concentration p_c' of ferromagnetic bonds for a non-Ising $\pm J$ model to have a ferromagnetic phase is not smaller than that of the Ising model p_c: $p_c' \geq p_c$. I also develop an argument that p_c is actually universal for any type of spin systems on any specified lattice, $p_c' = p_c$.

Let us assume that a bond takes a ferromagnetic exchange value J with probability p and an antiferromagnetic $-J$ with $1-p$. The lattice structure is arbitrary, and the range of interactions is also unspecified. For the Ising model with this type of interactions, it has been proved rigorously[9] that the ferromagnetic phase does not exist for $p<p_c$, where p_c is the point at which the Nishimori line defined by $\exp(-2J/k_BT)=(1-p)/p$ crosses the boundary between the paramagnetic and ordered phases as in Fig. 1. (Actually this point is believed to be the multicritical point where the paramagnetic, ferromagnetic and spin glass phases merge.[2-5]) The same type of argument applies as follows to show that no ferromagnetic phase can exist in the same region $p<p_c$ for a non-Ising $\pm J$ model with p_c being the critical concentration for the Ising model. This leads to the conclusion that the critical concentration p_c' for a non-Ising $\pm J$ model is larger than or equal to the Ising p_c.

To simplify the explanations, the $\pm J$ Heisenberg model is discussed hereafter. Generalizations are straightforward as explained below. Local gauge transformation leads to the following relation for the ferromagnetic correlation function,[9]

$$[\langle S_0 \cdot S_r \rangle_K]_p = [\langle \sigma_0 \sigma_r \rangle_{K_p} \langle S_0 \cdot S_r \rangle_K]_p, \qquad (1)$$

where the square brackets $[\cdots]_p$ denote the expectation value with respect to the $\pm J$ bond

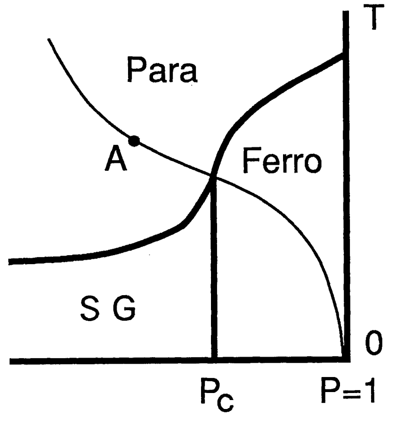

Fig. 1. The phase diagram of the $\pm J$ Ising model. The thin line is the Nishimori line. The ferromagnetic phase exists only for p larger than p_c with p_c being the value of p at the multicritical point. The $\pm JXY$, Heisenberg and other models are also expected to have a phase boundary parallel to the temperature axis at the same p_c which depends only on the lattice structure.

distribution, and the inner brackets $\langle\cdots\rangle_K$ express the thermal average for a given bond configuration with the absolute value of the coupling constant $K=J/k_BT$. The variables σ are Ising spins introduced by the gauge transformation. The thermal expectation value of these Ising variables on the right hand side of (1) is calculated at the coupling constant K_p defined by $\exp(-2K_p)=(1-p)/p$. The absolute value of the left hand side of (1) thus satisfies[9)]

$$|[\langle S_0\cdot S_r\rangle_K]_p| \leq [|\langle\sigma_0\sigma_r\rangle_{K_p}|]_p, \qquad (2)$$

since the absolute value of the correlation function $\langle S_0\cdot S_r\rangle_K$ does not exceed one under the normalization $|S_i|=1$. The right hand side of (2) is the spin glass correlation function evaluated on the Nishimori line of the $\pm J$ Ising model.[9)] Therefore, this quantity in the long-range limit $r\to\infty$ vanishes if the corresponding point on the Nishimori line is in the paramagnetic phase like the point A in Fig. 1. If p_c' for the Heisenberg model is smaller than the Ising p_c, it means that the Heisenberg model is in a ferromagnetic phase for $p_c'<p<p_c$. This contradicts with (2) because the right hand side vanishes for $r\to\infty$ in this range of p. Therefore we have $p_c'\leq p_c$. Note that a similar relation to (1) can be derived for any spin models (*e.g.*, $\pm JXY$ model) by gauge transformation if the spin space has a time reversal symmetry, $S_i\to -S_i$. This last condition excludes, for instance, the Potts model.

The argument in ref. 1 further went that the boundary line between the ferromagnetic and non-ferromagnetic (spin glass) phases of the $\pm J$ Ising model is parallel to the temperature axis as in Fig. 1. The reason was that, since the free energy on the Nishimori line can be interpreted as the entropy of distribution of frustration, the singularity in the free energy at the multicritical point (where $p=p_c$) implies a drastic change in the distribution of frustration on the underlying bond configuration, independently of thermal excitations. This geometrical singularity may manifest itself as singular behavior in thermodynamic quantities at p_c for any low temperature, and thus a vertical boundary emerges.

The above argument has been developed for the Ising model. However, the resulting singularity in the entropy of distribution of frustration is of purely geometrical nature in the sense that this singularity is a property of the bond configuration and has nothing to do with the spin variables. I used the $\pm J$ Ising model in deriving the singularity of the entropy of distribution of frustration, but the result can be interpreted genuinely in terms of the bond configuration. Therefore, any $\pm J$ models, Ising, XY, Heisenberg. etc., on the same lattice are likely to be affected by this same singularity in the geometrical structure of the underlying bond configuration. Thus it seems reasonable to expect the existence of a vertical phase boundary at the same concentration for any spin models, namely $p_c'=p_c$.

A recent numerical simulation of the $\pm J$ Heisenberg model on the simple cubic lattice indicates that the Heisenberg p_c' lies in the range 0.80 ± 0.01.[8)] The Ising model critical value is $p_c=0.77\pm0.01$.[3,5,10)] The rigorous inequality $p_c'\geq p_c$ is apparently satisfied. There is also a possibility that further detailed numerical investigations reveal that p_c' is actually equal to p_c as suggested above.

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