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Note on Twisted Order in the Frustrated Quantum Heisenberg Model

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Kishi and Kubo[1,2] showed rigorously that the two-dimensional quantum Heisenberg antiferromagnet with next nearest interactions does not have twisted[3-5] long-range order, defined by the root mean square of the order parameter in zero field, in the ground state. The Hamiltonian is

$$H_0=\sum_{\langle ij\rangle} \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\alpha \sum_{\langle lm\rangle} \boldsymbol{S}_l\cdot\boldsymbol{S}_m, \tag{1}$$

where the first summation is over nearest neighbor pairs and the second is over next nearest neighbors on the square lattice. Their proof applies to the parameter range $0\leq\alpha\leq 1/2$. We prove in the present note that the absence of spontaneous symmetry breaking corresponding to the twisted order follows from the Kishi-Kubo bound on the Duhamel two-point function (or the canonical correlation function) and the Bogoliubov inequality in a stronger form. One should remember that the absence of long-range order defined as above does not immediately imply the absence of spontaneous symmetry breaking, in particular if the order parameter does not commute with the Hamiltonian.[6)]

We add an external field to (1) to discuss the possibility of spontaneous symmetry breaking,

$$H_1=-h\sum_i \varepsilon_i \cos(\boldsymbol{k}\cdot\boldsymbol{i})S_i^y. \tag{2}$$

The factor ε_i assumes 1 if $\boldsymbol{i}$ is on one of the two sublattices of the Néel order and is -1 otherwise. Thus the symmetry breaking field corresponding to the ordinary Néel-type order has $\boldsymbol{k}=0$, while the twisted order is induced by $k_1\neq 0$ and $k_2=0$ (k_1 and k_2 are the first and second components, respectively, of $\boldsymbol{k}$). We thus assume $k_2=0$ in the following. The basic relation we use is the Bogoliubov inequality in a stronger form,[6)]

$$|\langle[A, B]\rangle|^2\leq\beta(B^\dagger, B)\langle[A^\dagger, [H, A]]\rangle, \tag{3}$$

where β is the inverse temperature and the brackets denote the Duhamel two-point function

$$(A, B)=Z^{-1}\int_0^1 \mathrm{Tr}\,\{\mathrm{e}^{-x\beta H}A\,\mathrm{e}^{-(1-x)\beta H}B\}\,\mathrm{d}x.$$

We choose A and B as follows:

$$A=S^z_{p-k+(\pi,\pi)}=\frac{1}{\sqrt{N}}\sum_j \varepsilon_j\,\mathrm{e}^{-\mathrm{i}(p-k)\cdot j}S_j^z,$$

and

$$B=S^x_{-p}=\frac{1}{\sqrt{N}}\sum_j \mathrm{e}^{\mathrm{i}p\cdot j}S_j^x.$$

The total number of sites is denoted by N. The commutator $[A, B]$ on the left-hand side of (3) is evaluated as

$$|\langle[A, B]\rangle|^2=\left\{\frac{1}{N}\sum_j \varepsilon_j\cos(\boldsymbol{k}\cdot\boldsymbol{j})\langle S_j^y\rangle\right\}^2+\left\{\frac{1}{N}\sum_j \varepsilon_j\sin(\boldsymbol{k}\cdot\boldsymbol{j})\langle S_j^y\rangle\right\}^2. \tag{4}$$

An upper bound on the Duhamel two-point function for the model (1) has been evaluated by Kishi and Kubo[1)] by the method of Dyson, Lieb and Simon.[6)] In applying the Kishi-Kubo argument to the present case H_0+H_1, it is important to note that the introduction of the field term H_1 of the form (2) does not invalidate the reflection symmetry if $\boldsymbol{k}$ is chosen parallel to the reflection axis, for instance $k_2=0$. The result of Kishi and Kubo remains intact for H_0+H_1:

$$\beta(S_p^x, S_{-p}^x)\leq B_p,$$

where

$$B_p=\frac{1}{2}\{2+\cos p_1+\cos p_2-2\alpha(1-\cos p_1\cos p_2)\}^{-1}, \tag{5}$$

which is valid only for $0\leq\alpha\leq 1/2$.[1)] Finiteness of (5) except for the case $p_1=p_2=\pi$ leads to the absence of long-range order of twisted type.[1,2)] A remarkable feature of the above expression (5) is that B_p does not depend upon the temperature. It means that the whole argument applies to any temperature including the ground state. The Bogoliubov inequality (3) is thus applicable to the ground-state problem if the double commutator of A and H can be evaluated in a form independent of the temperature. Another remark on (5) is that the right-hand side has no explicit h-dependence. The argument of Dyson, Lieb and Simon leading to the upper bound of the Duhamel two-point function, in their proof of Theorem 4.1, is unchanged if an external field is included in the Hamiltonian. Hence the manipulations of Kishi and Kubo leading to (5) need no modifications in the presence of a reflection-symmetric field.

It is straightforward to calculate the double commutator.

$$\begin{aligned}\langle[A^{\dagger}, [H, A]]\rangle = & -\frac{2}{N}\sum_{j,m}(1-\cos q_m)\{\langle S_j^x S_{j+m}^x\rangle + \langle S_j^y S_{j+m}^y\rangle\} \\ & -\frac{2\alpha}{N}\sum_j \{(1-\cos(q_1+q_2))(\langle S_j^x S_{j+1+2}^x\rangle + \langle S_j^y S_{j+1+2}^y\rangle) \\ & +(1-\cos(q_1-q_2))(\langle S_j^x S_{j+1-2}^x\rangle + \langle S_j^y S_{j+1-2}^y\rangle)\} + hm_y,\end{aligned}$$

where $\boldsymbol{q}=\boldsymbol{p}-\boldsymbol{k}+(\pi, \pi)$ (mod 2π). The first summation runs over all sites j and two spatial directions $m=1$ and 2. The subscript $\boldsymbol{j}+m$ represents the nearest neighbor site of $\boldsymbol{j}$ along the mth axis, and $\boldsymbol{j}+1+2$ and $\boldsymbol{j}+1-2$ denote two different next nearest neighbors. The magnetization m_y has been defined by

$$m_y = \frac{1}{N}\sum_j \varepsilon_j \cos(\boldsymbol{k}\cdot\boldsymbol{j})\langle S_j^y\rangle.$$

To estimate an upper bound on the double commutator, we replace correlation functions of the form $-\langle S_i^x S_j^x\rangle$ by their upper bound, S^2. The result is

$$\langle[A^{\dagger}, [H, A]]\rangle \leq 4S^2\{2-\cos q_1-\cos q_2+2\alpha(1-\cos q_1\cos q_2)\}+|hm_y| \equiv C_p. \tag{6}$$

With (5) and (6), the Bogoliubov inequality (3) reads

$$|\langle[A, B]\rangle|^2 \leq B_p C_p. \tag{7}$$

It is now possible to take the thermodynamic limit and the zero-temperature limit. After these limits, it is allowed to let the field h tend to zero. Since the left-hand side of (7) does not depend upon $\boldsymbol{p}$, see (4), the minimum of the right-hand side as a function of $\boldsymbol{p}$ gives an upper bound on the left-hand side. From the definition (6), C_p clearly vanishes at $q_1=q_2=0$ when $h=0$. Equation (5) indicates that B_p is finite except when $p_1=p_2=\pi$. Therefore, as $\boldsymbol{q}=\boldsymbol{p}-\boldsymbol{k}+(\pi, \pi)$ (mod 2π), the right-hand side of (7) vanishes unless $\boldsymbol{k}=0$. This implies $m_y=0$ according to (4). The Bogoliubov inequality (7) thus excludes the possibility of spontaneous symmetry breaking of twisted type ($k_1\neq 0$, $k_2=0$) which is sometimes considered as a candidate of non-classical ordering in the frustrated quantum system (1) when α is in the neighborhood of $1/2$.[3-5] The present argument applies to the region $0\leq \alpha\leq 1/2$, because the bound (5) is proved in this range.[1]

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