Lower critical dimension of the XY spin glass

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We develop an argument for the absence of spin-glass ordering in short-range vector spin systems with random symmetric exchange interactions in spatial dimension $d \le 4$. The present argument supplements our previous theory; we consider the possibility of divergence in some correlation functions in the limit of small external field. It is also shown that the comment of Schwartz and Young on our previous theory is inappropriate.

I. INTRODUCTION

It is now widely believed from numerical evidence[1-9] that the short-range vector-spin models do not undergo a conventional spin-glass transition at a finite temperature if the spatial dimensionality d is less than 4. It is thus desirable to determine analytically the lower critical dimension of vector spin glasses.[6,10-14] In our previous paper,[13] we argued that the spin-glass order parameter vanishes if $d \le 4$ for the short-range vector spin system using the Mermin-type argument[15] for ferromagnets with the replica method by assuming clustering properties of spin-correlation functions. We implicitly supposed there that, if clustering properties hold, some correlation functions do not diverge, such as the transverse spin-glass susceptibility which consists of spin components perpendicular to the applied random field. However, there is actually a possibility of divergence of such correlation functions in the limit of small field as discussed in the present paper, although such divergences do not affect our conclusion. Moreover, Schwartz and Young[14] claimed to have shown that the assumption of clustering properties is not satisfied. They argued that the external random field we applied does not break the symmetry completely, which is the origin of failure of our argument. This problem is closely related with the above-mentioned divergence of correlation functions. We show in the present paper that their argument is incorrect and our previous conclusion remains valid even if the possibility of divergence is considered.

This paper is organized as follows. In Sec. II the model studied is defined and the Schwarz inequality, from which the absence of ordering is concluded, is introduced to this model. The size dependence of correlation functions appearing in the Schwarz inequality is discussed in Sec. III, assuming the clustering property. In Sec. IV, we conclude the absence of spin-glass ordering for $d \le 4$ by considering the possibility of divergence of some correlation functions. Section V is devoted to discussions.

II. DEFINITIONS AND BASIC INEQUALITY

For simplicity, we restrict our attention to the plane rotator model on a d-dimensional hypercubic lattice with external random fields. Anisotropies considered in Ref. 13 are omitted: No essential change is necessary when one generalizes the following argument to a model with such anisotropies. The Hamiltonian is given by

$$\mathcal{H} = \mathcal{H}_{\rm ex} + \mathcal{H}_f \ , \qquad (2.1)$$

$$\mathcal{H}_{\rm ex} = -\sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j) \ , \qquad (2.2)$$

$$\mathcal{H}_f = -h \sum_i \xi_i \cos\theta_i \ . \qquad (2.3)$$

The range of interaction is short. The coupling J_{ij} is a quenched random variable with the symmetric Gaussian distribution of variance J^2. The term $\mathcal{H}_f$ represents a symmetry-breaking random field along one axis. The distribution of ξ_i is assumed to be symmetric Gaussian with variance unity. Using the replica method, we derive the effective Hamiltonian $\mathcal{H}'(n)$ defined by

$$\mathrm{Tr} \exp[-\beta \mathcal{H}'(n)] \equiv [Z(\{J_{ij}\},\{\xi_i\})^n]_c \ . \qquad (2.4)$$

The square brackets $[\ ]_c$ denote the configurational average over the distributions of $\{J_{ij}\}$ and $\{\xi_i\}$ and Tr is for the integration over the variables $\{\theta_i^\alpha\}$, where $\alpha(=1,\ldots,n)$ is the replica index. After performing the configurational average, one finds

$$\mathcal{H}'(n) = \mathcal{H}'_{\rm ex}(n) + \mathcal{H}'_f(n) \ , \qquad (2.5)$$

$$\mathcal{H}'_{\rm ex}(n) = -\frac{\beta}{2} J^2 \sum_{\langle ij \rangle} \left\{ \sum_\alpha \cos(\theta_i^\alpha - \theta_j^\alpha) \right\}^2 \ , \qquad (2.6)$$

$$\mathcal{H}'_f(n) = -\frac{\beta}{2} h^2 \sum_i \left\{ \sum_\alpha \cos\theta_i^\alpha \right\}^2 \ . \qquad (2.7)$$

The spin-glass order parameter q is defined as

$$q = \lim_{h\to 0} q(h) = \lim_{h\to 0} \lim_{N\to\infty} \lim_{n\to 0} q(h,N,n) \ , \qquad (2.8)$$

where N is the system size and

$$q(h,N,n) = \langle \cos\theta_i^\gamma \cos\theta_i^\delta \rangle \quad (\gamma \ne \delta) \ . \qquad (2.9)$$

Here the angular brackets represent the thermal average over the replicated system:

$$\langle\ \rangle = \mathrm{Tr} \cdots \exp[-\beta \mathcal{H}'(n)] \ . \qquad (2.10)$$

In order to discuss the absence of ordering, we make use of the Schwarz inequality

$$\langle A^*A\rangle \geq \frac{|\langle A^*B\rangle|^2}{\langle B^*B\rangle}, \tag{2.11}$$

and choose A and B as

$$A=\sum_j \exp[-i\mathbf{k}\cdot\mathbf{R}_j]\sin\theta_j^\gamma \sin\theta_j^\delta \quad (\gamma\neq\delta), \tag{2.12}$$

$$B=-\sum_{l,m} \exp[-i\mathbf{k}\cdot(\mathbf{R}_l+\mathbf{R}_m)/2]\frac{\partial^2\mathcal{H}'(n)}{\partial\theta_l^\gamma\partial\theta_m^\delta} \quad (\gamma\neq\delta). \tag{2.13}$$

III. SIZE DEPENDENCE OF CORRELATION FUNCTIONS

In this section, we discuss the size dependence of correlation functions appearing in Eq. (2.11) because, in the next section, we make use of the inequality (2.11) in the thermodynamic limit $N\to\infty$ with h kept finite in order to discuss the absence of spin-glass ordering. Let us substitute Eq. (2.5) into Eq. (2.13) to obtain

$$B=B_{\rm ex}+\beta h^2 A, \tag{3.1}$$

where $B_{\rm ex}$ is defined by Eq. (2.13) with $\mathcal{H}'(n)$ replaced by $\mathcal{H}'_{\rm ex}(n)$. Thus, one has

$$\langle A^*B\rangle=\langle A^*B_{\rm ex}\rangle+\beta h^2\langle A^*A\rangle \tag{3.2}$$

and also

$$\langle B^*B\rangle=\langle B^*_{\rm ex}B_{\rm ex}\rangle+2\beta h^2{\rm Re}(\langle A^*B_{\rm ex}\rangle)+\beta^2h^4\langle A^*A\rangle, \tag{3.3}$$

where Re( ) denotes the real part. Taking the limits of $n\to0$ and $N\to\infty$, we obtain

$$\lim_{N\to\infty}\lim_{n\to0}\frac{1}{N}\langle B^*_{\rm ex}B_{\rm ex}\rangle=\frac{1}{4}\beta^2J^4\sum_{\Delta,\Delta'}\tilde{\phi}_1(\mathbf{k},\boldsymbol{\Delta},\boldsymbol{\Delta}')(1-\exp[i\mathbf{k}\cdot\boldsymbol{\Delta}/2])^2(1-\exp[-i\mathbf{k}\cdot\boldsymbol{\Delta}'/2])^2, \tag{3.4a}$$

$$\lim_{N\to\infty}\lim_{n\to0}\frac{1}{N}\langle A^*B_{\rm ex}\rangle=\frac{1}{2}\beta J^2\sum_{\Delta}\tilde{\phi}_2(\mathbf{k},\Delta)(1-\exp[i\mathbf{k}\cdot\Delta/2])^2, \tag{3.4b}$$

$$\lim_{N\to\infty}\lim_{n\to0}\frac{1}{N}\langle A^*A\rangle=\tilde{\phi}_3(\mathbf{k}), \tag{3.4c}$$

$$\tilde{\phi}_1(\mathbf{k},\boldsymbol{\Delta},\boldsymbol{\Delta}')=\sum_{\mathbf{R}}\exp[i\mathbf{k}\cdot\mathbf{R}]\phi_1(\mathbf{R},\boldsymbol{\Delta},\boldsymbol{\Delta}'), \tag{3.5a}$$

$$\tilde{\phi}_2(\mathbf{k},\boldsymbol{\Delta})=\sum_{\mathbf{R}}\exp[i\mathbf{k}\cdot\mathbf{R}]\phi_2(\mathbf{R},\Delta), \tag{3.5b}$$

$$\tilde{\phi}_3(\mathbf{k})=\sum_{\mathbf{R}}\exp[i\mathbf{k}\cdot\mathbf{R}]\phi_3(\mathbf{R}), \tag{3.5c}$$

$$\phi_1(\mathbf{R}_l-\mathbf{R}_m,\boldsymbol{\Delta},\boldsymbol{\Delta}')=\lim_{N\to\infty}[\langle\sin(\theta_l-\theta_{l+\Delta})\sin(\theta_m-\theta_{m+\Delta'})\rangle_T^2]_c, \tag{3.6a}$$

$$\phi_2(\mathbf{R}_l-\mathbf{R}_m,\boldsymbol{\Delta})=\lim_{N\to\infty}[\langle\sin\theta_l\sin(\theta_m-\theta_{m+\Delta})\rangle_T^2]_c, \tag{3.6b}$$

$$\phi_2(\mathbf{R}_l-\mathbf{R}_m)=\lim_{N\to\infty}[\langle\sin\theta_l\sin\theta_m\rangle_T^2]_c, \tag{3.6c}$$

where $\boldsymbol{\Delta}$ and $\boldsymbol{\Delta}'$ are nearest-neighbor vectors. The brackets $\langle\ \rangle_T$ denote the thermal average over the nonreplicated system with one particular bond configuration Eq. (2.1); in Eqs. (3.6), the limits of $n\to0$ have already been taken, so that one can express ϕ's by the thermal and configurational averages instead of the replica representations.[16] We assume here the clustering properties for some correlation functions such as $\phi_3(\mathbf{R})$ under the external random field of Eq. (2.3). This means that $\phi_3(\mathbf{R})$ is short ranged or decays exponentially as $|\mathbf{R}|\to\infty$, and $\tilde{\phi}_3(\mathbf{k})$ remains finite as $|\mathbf{k}|\to0$. Then $\langle A^*A\rangle$ is of $O(N)$. A similar argument can be made for $\langle B^*_{\rm ex}B_{\rm ex}\rangle$ and $\langle A^*B_{\rm ex}\rangle$ when appropriate clustering properties are assumed. Therefore, from Eqs. (3.2) and (3.3), $\lim_{N\to\infty}\lim_{n\to0}\langle A^*A\rangle/N$, $\lim_{N\to\infty}\lim_{n\to0}\langle B^*B\rangle/N$, and $\lim_{N\to\infty}\lim_{n\to0}\langle A^*B\rangle/N$ give finite values.

Since we investigate the infrared (small-k) divergence of the integration of the right-hand side (rhs) of Eq. (2.11) in the limit of small field $h\to0$ in Sec. IV, it is sufficient to consider only the leading contribution in the variable $k\equiv|\mathbf{k}|$ in $\langle A^*B\rangle$ and $\langle B^*B\rangle$. Using Eq. (3.3) together with Eqs. (2.5)–(2.7), (2.12) and (2.13), we obtain[13]

$$\lim_{N\to\infty}\lim_{n\to0}\frac{1}{N}\langle B^*B\rangle\simeq c_1k^4+c_2k^2h^2+c_3h^4, \tag{3.7}$$

with

$$c_1=\lim_{k\to0}\lim_{N\to\infty}\lim_{n\to0}\frac{1}{Nk^4}\langle B^*_{\rm ex}B_{\rm ex}\rangle =\frac{1}{64}\beta^2J^4\sum_{\Delta,\Delta'}\Delta_{\mathbf{k}}^2\Delta_{\mathbf{k}}'^2\tilde{\phi}_1(0,\boldsymbol{\Delta},\boldsymbol{\Delta}'), \tag{3.8}$$

$$c_2=\lim_{k\to0}\lim_{N\to\infty}\lim_{n\to0}\frac{1}{Nk^2}2\beta\langle A^*B_{\rm ex}\rangle =-\frac{1}{4}\beta^2J^2\sum_{\Delta}\Delta_{\mathbf{k}}^2\tilde{\phi}_2(0,\boldsymbol{\Delta}), \tag{3.9}$$

$$c_3=\lim_{k\to0}\lim_{N\to\infty}\lim_{n\to0}\frac{1}{N}\beta^2\langle A^*A\rangle =\beta^2\tilde{\phi}_3(0), \tag{3.10}$$

where $\Delta_{\mathbf{k}}$ and $\Delta'_{\mathbf{k}}$ are the projections of nearest-neighbor vectors $\mathbf{\Delta}$ and $\mathbf{\Delta}'$ to the direction along $\mathbf{k}$, respectively, which are of the order of the lattice constant. From Eqs. (3.8)–(3.10), it is clear that c_1 and c_3 are real and positive and c_2 is real and negative. Note that c_3 is the transverse spin-glass susceptibility, which consists of spin components perpendicular to the applied random field.

There is an alternative expression of $\langle A^*B\rangle$ to be compared with Eq. (3.2). Integrating by parts, we find[13]

$$\langle A^*B\rangle=\beta^{-1}Nq(h,N,n) -\beta\sum_{l,m}\exp[-i\mathbf{k}\cdot(\mathbf{R}_l+\mathbf{R}_m)/2] \times\left\langle A^*\frac{\partial\mathcal{H}'(n)}{\partial\theta_l^\gamma}\frac{\partial\mathcal{H}'(n)}{\partial\theta_m^\delta}\right\rangle . \quad (3.11)$$

It is easy to see that

$$\lim_{k\to 0}\sum_{l,m}\exp[-i\mathbf{k}\cdot(\mathbf{R}_l+\mathbf{R}_m)/2]\left\langle A^*\frac{\partial\mathcal{H}'(n)}{\partial\theta_l^\gamma}\frac{\partial\mathcal{H}'(n)}{\partial\theta_m^\delta}\right\rangle =\sum_{l,m}\left\langle A^*\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_l^\gamma}\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_m^\delta}\right\rangle . \quad (3.12)$$

In the limit of $k\to 0$, we obtain, by identifying Eqs. (3.2) and (3.11), with Eqs. (3.4) and (3.12) taken into account,

$$\lim_{k\to 0}\beta h^2\langle A^*A\rangle=\beta^{-1}Nq(h,N,n) -\lim_{k\to 0}\beta\sum_{l,m}\left\langle A^*\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_l^\gamma}\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_m^\delta}\right\rangle . \quad (3.13)$$

Since the first and second terms in Eq. (3.2) are calculated by Eqs. (3.9) and (3.13), respectively, we find

$$\lim_{N\to\infty}\lim_{n\to 0}\frac{1}{N}\langle A^*B\rangle\simeq\tfrac{1}{2}\beta^{-1}c_2k^2+\beta^{-1}q(h)-c_4h^4 , \quad (3.14)$$

with

$$c_4=\lim_{N\to\infty}\lim_{n\to 0}\lim_{k\to 0}\frac{1}{Nh^4}\beta\sum_{l,m}\left\langle A^*\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_l^\gamma}\frac{\partial\mathcal{H}'_f(n)}{\partial\theta_m^\delta}\right\rangle =\beta^3\lim_{N\to\infty}\frac{1}{N}\sum_{j,l,m}\phi_4(\mathbf{R}_j,\mathbf{R}_l,\mathbf{R}_m) , \quad (3.15)$$

$$\phi_4(\mathbf{R}_j,\mathbf{R}_l,\mathbf{R}_m)=\lim_{n\to 0}\sum_{\mu,\nu}\langle\sin\theta_j^\gamma\sin\theta_j^\delta\sin\theta_l^\gamma \times\cos\theta_l^\mu\sin\theta_m^\delta\cos\theta_m^\nu\rangle . \quad (3.16)$$

In Eq. (3.16), we have not used an expression after taking the limit of $n\to 0$ as in Eqs. (3.8)–(3.10), because the resulting formula is complicated and not very useful. All c's are functions of β and h independent of k and N. Since $\lim_{N\to\infty}\lim_{n\to 0}\langle A^*B\rangle/N$ is of order unity if the clustering properties hold, so is the rhs of Eq. (3.14) irrespective of k. Of course $q(h)$ is also of order unity. Therefore, although c_4 is defined by a summation of three-point correlation functions, it is also of order unity assuming only the clustering properties of two-point correlation functions.

IV. ABSENCE OF ORDERING

Let us now show that the inequality (2.11) leads to the absence of spin-glass ordering. The following relation will be used later:

$$N\geq\frac{1}{N}\sum_{\mathbf{k}}\langle A^*A\rangle , \quad (4.1)$$

which can be derived in a straightforward way. Substituting Eqs. (3.7) and (3.14) into Eq. (2.11) and summing over all $\mathbf{k}$ in the first Brillouin zone, with Eq. (4.1) taken into account, we obtain

$$1\geq[\beta^{-1}q(h)-c_4h^4]^2I(h) , \quad (4.2)$$

$$I(h)=\int\frac{d\mathbf{k}}{c_1k^4+c_2k^2h^2+c_3h^4} , \quad (4.3)$$

where an irrelevant term in Eq. (3.14) has been dropped; the argument remains almost the same if the first term on the rhs of Eq. (3.14), proportional to c_2k^2, is included. It is clear that $I(h)$ diverges if $d\leq 4$ for $h\to 0$. In fact, we have proved[13] that, if all c's are finite, the integral $I(h)$ diverges for $d\leq 4$ in the limit of small field. This implies $q(h)\to 0$ as $h\to 0$ if $d\leq 4$.

The above argument is based upon the assumption of the clustering property of some correlation functions. It breaks down if the spin-correlation functions in Eqs. (3.8)–(3.10) and (3.15) do not cluster in a sufficiently large system. There is another possible breakdown of this argument. Using Eqs. (3.13), (3.10), and (3.15), we find an important identity

$$q(h)=c_3h^2+c_4h^4\beta . \quad (4.4)$$

From Eq. (4.4), c_3 and/or c_4 diverge in the limit of small field if $q\neq 0$. This means the divergence of the transverse spin-glass susceptibility in the limit of small field. The fluctuation perpendicular to the external field h grows infinitely as h tends to zero. The same situation occurs in nonrandom ferromagnetic systems (see Appendix A). The divergence of c_3h^4 as $h\to 0$ causes the divergence of the last term in the denominator of Eq. (4.3). If c_4h^4 at $h=0$ is nonzero, we can derive nothing for the order parameter q from Eq. (4.2) even if $I(h)$ diverges. In these cases the above argument for $q=0$ if $d\leq 4$ breaks down. Therefore, it is necessary to consider such divergences in the coefficients c_j for $h\to 0$ to firmly conclude the absence of spin-glass ordering. The behaviors of c_3 and c_4 relate to each other through Eqs. (4.2) and (4.4). If, for instance, c_3h^2 diverges as $h\to 0$, c_4h^4 should diverge with the same order as c_3h^2 from Eq. (4.4), since the order parameter $q(h)$ does not diverge because of the definition Eq. (2.9).

Let us first point out that the divergences in c_2 and c_4 come from the growth of fluctuations in spin-glass correlations perpendicular to the applied field, as seen from the fact that c_3 is the transverse spin-glass susceptibility. The correlations containing the factor like $\sin(\theta_l^\gamma-\theta_j^\gamma)$, as in ϕ_1 and ϕ_2, are not related to this type of transverse instability and, therefore, c_1 and c_2 are not expected to grow indefinitely; the correlation functions including the differences of nearest-neighboring angles describe fluctuations of non-spin-glass-type operators. (Note that the chirality spin-glass order parameter has more complicated contributions of sine functions.[9]) Therefore, it is physically reasonable to assume that the divergence does not occur in c_1 and c_2.

Let us introduce the leading power of h in c_3:

$$c_3 \sim h^{-\epsilon} . \tag{4.5}$$

In Appendix B, we show the inequality

$$c_4 \le (c_3)^2 , \tag{4.6}$$

which is valid as long as c_3 diverges as $h \to 0$; only the leading contributions have been compared except for prefactors in deriving Eq. (4.6). Furthermore, in Appendix B, we show that

$$c_4 = (c_3)^2 , \tag{4.7}$$

in low-temperature regions where $q \neq 0$, using a finite-size-scaling argument. In the case of $\epsilon > 2$, $c_3 h^2$ diverges as $h \to 0$, so does $c_4 h^4$ with the same order of $c_3 h^2$ because of Eq. (4.4); $q(h)$ does not diverge. This contradicts the fact that $c_4 h^4 \sim (c_3)^2 h^2$ from Eq. (4.7). Therefore, even if c_3 diverges as $h \to 0$, the power ϵ must be less than or equal to 2 in any dimension. Next, we show that the power ϵ must be less than 2 if $d \le 4$. In the case of $\epsilon = 2$, $c_3 h^2$ gives a finite value at $h = 0$. Since $c_3 h^4$ behaves as h^2 and vanishes as $h \to 0$, $I(h)$ in Eq. (4.3) diverges if $d \le 4$. Therefore, we find that $[q(h) - c_4 h^4]^2 I(h)$, which is equal to $(c_3 h^2)^2 I(h)$ from Eq. (4.4), diverges for $\epsilon = 2$ if $d = 4$, which contradicts Eq. (4.2).

Consequently, it is concluded from Eqs. (4.2) and (4.4) that, even if c_3 diverges as $h \to 0$, its power is less than 2 when $d \le 4$. Thus, both $c_3 h^4$ in Eq. (4.3) and $c_3 h^2$ in Eq. (4.4) vanish, which indicate $[\beta^{-1} q(h) - c_4 h^4]^2 \to 0$. Clearly, $c_4 h^4$ does not diverge from Eq. (4.4). Since $c_3 h^2$ vanishes as $h \to 0$, so does $c_4 h^4$ from Eq. (4.6), which means the absence of spin-glass ordering $q = 0$ because of Eq. (4.4). Note that the above conclusion is derived for $d \le 4$. If $d > 4$, the possibility $\epsilon = 2$ cannot be excluded, and we cannot proceed further within this framework.

V. DISCUSSION AND CONCLUSION

We have studied the short-range vector-spin-glass model and shown that the lower bound of the lower critical dimension is 4 from the fact that $q = 0$ if $d \le 4$. Although we treated only the plane rotator model with no anisotropies, the conclusion of the absence of ordering in the XY plane can easily be extended to a model with some anisotropies.[11,13] Our argument is based upon the assumption of the clustering properties of some correlation functions, which was necessary to show the finiteness of the functions $c_1, c_2, \ldots$. Therefore, the above conclusion is restricted to the case in which the low-temperature phase does not have a multivalley structure, because spin-correlation functions do not cluster in the phase space with the multivalley structure.[17] We pointed out that, even if all correlation functions cluster, there is a possibility of divergence in c_3 and c_4 in the limit of small field after the thermodynamic limit is taken. The above conclusion has been derived by taking into account this possibility.

The criticism of Schwartz and Young[14] on our previous paper[13] is based upon comparison of two expressions of $\langle A^* B \rangle$, Eqs. (3.2) and (3.11). They claim to have shown that when $h = 0$, $\tilde{\phi}_2(\mathbf{k}, \Delta)$ diverges as $k \to 0$ if $q \neq 0$. This means the breakdown of the assumption of the clustering property. Here we consider these two expressions of $\langle A^* B \rangle$ in detail to confirm the consistency of assuming the clustering properties. If Schwartz and Young derived their conclusion by setting h to zero before taking the thermodynamic limit, then, from the definition of the order parameter Eq. (2.9), $q(h,N,n)$ is identically equal to zero because of the rotation symmetry in the Hamiltonian of finite systems. Therefore, their argument does not make sense. In this case, the second term on the rhs of Eq. (3.2) and the first term in Eq. (3.11) vanish, and we obtain by identifying Eqs. (3.2) and (3.11)

$$\langle A^* B_{\text{ex}} \rangle = \sum_{l,m} \exp[-i\mathbf{k}\cdot(\mathbf{R}_l + \mathbf{R}_m)/2] \times \left\langle A^* \frac{\partial \mathcal{H}'(n)}{\partial \theta_l^\gamma} \frac{\partial \mathcal{H}'(n)}{\partial \theta_m^\delta} \right\rangle . \tag{5.1}$$

In the limit of $k \to 0$, it can easily be seen that both sides vanish as k^2. Therefore, no difficulties arise. If, on the other hand, they implied that h is set to zero after taking the thermodynamic limit, their argument, that the correlation function $\phi_2(\mathbf{R}, \Delta)$ does not cluster if there exists finite ordering, does not conflict with our conclusion; we derived the absence of ordering by assuming clustering properties, which is contrapositive to their statement.

Schwartz and Young also pointed out that, while the present random field (2.3) breaks the degeneracy between thermodynamic states related by the symmetry of uniform rotation like $\theta_i^\alpha \to \theta_i^\alpha + \delta\theta$ for all i, it does not distinguish between states related by reflection symmetry like $\theta_i^\alpha \to -\theta_i^\alpha$ for all i. Such symmetry breakdown could occur only if the low-temperature phase has a nontrivial multivalley structure. As we remarked in Ref. 13, our argument breaks down if the low-temperature phase has a mean-field-like multivalley structure because, on this structure, the spin-correlation functions do not cluster in a sufficiently large system. We thus made an explicit statement there that the existence of an ordered phase with a multivalley structure is not excluded on the basis of our argument. What we have shown is that the reflection-symmetric spin-glass phase defined by the nonvanishing value of q, Eqs. (2.8) and (2.9), does not exist if $d \le 4$. The possibility of a reflection-nonsymmetric mean-field-like phase must be investigated by other methods.

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APPENDIX A: DIVERGENCE OF THE TRANSVERSE SUSCEPTIBILITY

The function c_3 in Eq. (3.10) is proportional to the transverse spin-glass susceptibility. It is not surprising that the transverse part of the response function diverges in the limit of small field. Here we see how this phenomenon occurs in the nonrandom ferromagnetic XY model. The Hamiltonian is

$$\mathcal{H}^{(f)}=\mathcal{H}^{(f)}_{\rm ex}+\mathcal{H}^{(f)}_{f} , \tag{A1}$$

$$\mathcal{H}^{(f)}_{\rm ex}=-J\sum_{\langle ij\rangle}\cos(\theta_i-\theta_j) , \tag{A2}$$

$$\mathcal{H}^{(f)}_{f}=-h\sum_{i}\cos\theta_i . \tag{A3}$$

Let us consider the correlation function

$$C(h)=\left\langle \sum_{l,m}\sin\theta_l\frac{\partial\mathcal{H}^{(f)}}{\partial\theta_m}\right\rangle , \tag{A4}$$

which corresponds to $\langle A^*B\rangle$ with $\mathbf{k}=0$ in Eq. (2.11). Similarly to the random case in Sec. III, $C(h)$ can be expressed in two distinct ways. First, we obtain

$$C(h)=\beta^{-1}\sum_{l}\langle\cos\theta_l\rangle$$
$$\equiv\beta^{-1}Nm(h) , \tag{A5}$$

by integrating by parts, where $m(h)$ is the magnetization of the longitudinal component. Second, we have

$$C(h)=h\sum_{l,m}\langle\sin\theta_l\sin\theta_m\rangle$$
$$\equiv\beta^{-1}Nh\chi_T(h) , \tag{A6}$$

by substituting Eqs. (A1)–(A3) into Eq. (A4), where $\chi_T(h)$ is the susceptibility of the transverse component; the term $\langle\sin\theta\rangle^2$ is lacking in Eq. (A6) because it is identically equal to zero when $h\neq0$. Then we find

$$m(h)=h\chi_T(h) . \tag{A7}$$

Since there is no explicit dependence on the size N in Eq. (A7), we may understand that this equation holds in the thermodynamic limit. If the temperature T is above the ferromagnetic critical point T_c, $\chi_T(h)$ is finite for any values of h. On the other hand, if $T\le T_c$, $\chi_T(h)$ behaves as

$$\chi_T(h)\sim\begin{cases}h^{-1+1/\delta} & (T=T_c)\\ h^{-1} & (T<T_c) ,\end{cases} \tag{A8}$$

indicating the divergence as $h\to0$. Therefore, anomalous transverse fluctuations are observed in the limit of small longitudinal fields when the order parameter is finite. This suggests an infinite growth of the correlation length associated with the transverse components

$$\xi_T\equiv\lim_{r\to\infty}\lim_{N\to\infty}\left|\frac{d}{dr}\ln\langle\sin\theta_0\sin\theta_r\rangle\right|^{-1} . \tag{A9}$$

APPENDIX B: RELATION BETWEEN THE COEFFICIENTS c_3 AND c_4

Here we derive the relation between c_3 and c_4 when they diverge. Using ϕ_3 in Eq. (3.6c) and ϕ_4 in Eq. (3.16), we can approximately express c_3 and c_4 as

$$c_3=\beta^2\int\phi_3(\mathbf{r})d\mathbf{r} , \tag{B1}$$

$$c_4=\beta^2\int\int\phi_4(0,\mathbf{r},\mathbf{r}')d\mathbf{r}\,d\mathbf{r}' . \tag{B2}$$

Both $\phi_3(\mathbf{r})$ and $\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ decay to zero in the asymptotic region because of the clustering property. Since $\phi_3(\mathbf{r})$ is a spin-glass correlation function, it is natural to assume that the function $\phi_3(\mathbf{r})$ is the slowest-decaying correlation function among all two-point correlation functions in the spin-glass critical regime, the region of small field in the present case. More precisely, the correlation length determined from $\phi_3(\mathbf{r})$

$$\xi_{SG}\equiv\lim_{r\to\infty}\left|\frac{\partial}{\partial r}\ln\phi_3(\mathbf{r})\right|^{-1} \tag{B3}$$

is regarded as the spin-glass correlation length, which is the longest characteristic length at this regime. If only one of three arguments (say, $|\mathbf{r}_1|$) in $\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ is large and the others ($|\mathbf{r}_2|$ and $|\mathbf{r}_3|$) are finite, we may regard the correlation function $\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ as one of the two-point correlation functions, and we obtain

$$\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\le\phi_3(\mathbf{r}_1) . \tag{B4}$$

Similar relations hold as

$$\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\le\phi_3(\mathbf{r}_2) \tag{B5}$$

for $|\mathbf{r}_1|$ and $|\mathbf{r}_3|$ being finite, and

$$\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\le\phi_3(\mathbf{r}_3) \tag{B6}$$

for $|\mathbf{r}_1|$ and $|\mathbf{r}_2|$ being finite. If two of three arguments (say, $|\mathbf{r}_2|$ and $|\mathbf{r}_3|$) in $\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ are large and the other is finite (set $|\mathbf{r}_1|=0$), it is reasonable to estimate the upper bound of $\phi_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ as

$$\phi_4(0,\mathbf{r},\mathbf{r}')\le\phi_3(\mathbf{r})\phi_3(\mathbf{r}') , \tag{B7}$$

where the leading contributions in the asymptotic region are compared. Of course, inequality (B7) is consistent with inequalities (B4)–(B6) apart from prefactors. Substituting Eq. (B7) into Eq. (B2) and using Eq. (B1), we obtain

$$c_4\le(c_3)^2 . \tag{B8}$$

Let us denote the finite-volume ($N=L^d$) version of c_3 and c_4 as $c_3(L)$ and $c_4(L)$, which are defined by Eqs. (3.10) and (3.15) without taking the limit of $N\to\infty$. If there exists a finite spin-glass ordering at $h=0$, $\phi_3(\mathbf{r})$ and $\phi_4(0,\mathbf{r},\mathbf{r}')$ do not vanish in the asymptotic regime ($|\mathbf{r}|\to\infty$ and/or $|\mathbf{r}'|\to\infty$). Then one may expect that $c_3(L)$ is $O(N)$ from Eq. (3.10) and c_4 is $O(N^2)$ from Eq. (3.15). We introduce the finite-size-scaling form of $c_3(L)$ and $c_4(L)$ as

$$c_3(L)=L^d f(L^y h^2) , \tag{B9}$$

$$c_4(L)=L^{2d} g(L^y h^2) , \tag{B10}$$

where f and g are analytic functions satisfying

$$f(x)=\begin{cases}\text{const} & (x \ll 1)\\ x^{-d/y} & (x \gg 1) ,\end{cases} \tag{B11}$$

and

$$g(x)=\begin{cases}\text{const} & (x \ll 1)\\ x^{-2d/y} & (x \gg 1) .\end{cases} \tag{B12}$$

We assume that the exponents y's in Eqs. (B9) and (B10) are common. (In a nonrandom ferromagnetic system, it is believed[18] that $y=d/2$ if $T<T_c$.) From Eqs. (B9)–(B12), we find

$$c_4=(c_3)^2 \tag{B13}$$

in the limit $L \to \infty$.

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