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Gauge Glass Ordering in Two Dimensions

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We prove that there is no gauge glass ordering in two dimensions at finite temperatures. Numerical results have thus been rigorously justified.

[gauge glass, Schwarz inequality, long-range order]

Gauge glass is a simplified model to describe the vortex glass state of type-II superconductors with impurities.[1,2] The problem of existence or absence of finite gauge glass ordering has been investigated mainly by numerical methods.[3-8] The current general agreement is that the short-range three-dimensional system may have gauge glass ordering at finite temperatures while the two-dimensional counterpart does not.

We show in this paper that a part of the above results can be proved rigorously: There is no gauge glass ordering in two dimensions at finite temperatures. The proof is similar to the Mermin argument[9] for the absence of regular ordering in the non-random XY model. The only point of modification from the Mermin argument is to introduce two real replicas so that glass-type ordering can be treated in a random system.

Let us first state the result explicitly. The Hamiltonian is

$$H=-\sum_{\langle ij\rangle}\{\cos(\theta_i^{(1)}-\theta_j^{(1)}+A_{ij})+\cos(\theta_i^{(2)}-\theta_j^{(2)}+A_{ij})\}-h\sum_i\cos(\theta_i^{(1)}-\theta_i^{(2)}),\qquad(1)$$

where $\theta_i^{(k)}$ $(k=1,2)$ denotes the dynamical degree of freedom of the kth replica. The last term in (1) is necessary to force the two replicas to fall in the same valley in the energy landscape.[10-14]*

The quenched random variable A_{ij} is fixed arbitrarily at each pair of sites $\langle ij\rangle$. The summation in (1) runs over short-range pairs on a d-dimensional lattice. We prove below that the gauge glass order parameter

$$q=\langle\exp\{\mathrm{i}(\theta_i^{(1)}-\theta_i^{(2)})\}\rangle\qquad(2)$$

approaches zero as $h\to 0$ if $d\leq 2$ and $T>0$. The angular brackets in (2) represent the thermal expectation value with respect to the Hamiltonian (1) with arbitrarily fixed $\{A_{ij}\}$. The order parameter (2) measures similarity of states in two replicas and thus serves as an order parameter of glass-type ordering.[10-15]

The proof consists of a simple application of the Schwarz inequality $\langle AA^*\rangle\geq|\langle AB^*\rangle|^2/\langle BB^*\rangle$. We choose A and B as

$$A=\frac{1}{\sqrt{N}}\sum_j\exp(-\mathrm{i}kR_j)\exp(\mathrm{i}\psi_j),$$
$$B=\frac{1}{\sqrt{N}}\sum_j\exp(-\mathrm{i}kR_j)\frac{\partial H}{\partial\psi_j},\qquad(3)$$

where ψ_j denotes the difference $\theta_j^{(1)}-\theta_j^{(2)}$. Independent variables will be $\theta_j^{(1)}$ and ψ_j hereafter. By integrating by parts, we find

$$\langle AB^*\rangle=\mathrm{i}k_B Tq$$

and

* We use the technique appearing in the first half of ref. 11. We refer the reader to ref. 15 for detailed discussions on validity of the latter half of the arguments in ref. 11.

$$\langle BB^*\rangle=\frac{2k_{\rm B}T}{N}\sum_{\langle ij\rangle}\{1-\cos k(R_i-R_j)\}$$
$$\times\langle\cos(\theta_i^{(1)}-\theta_j^{(1)}+\psi_i-\psi_j+A_{ij})\rangle$$
$$+hk_{\rm B}T\,{\rm Re}\,(q)\leq k_{\rm B}T(ck^2+h),$$

where c is a positive constant and Re represents the real part. Using (3) for the explicit expression of $\langle AA^*\rangle$ and integrating both sides of the Schwarz inequality over the first Brillouin zone, we obtain

$$1\geq k_{\rm B}Tq^2\int\frac{{\rm d}^d k}{ck^2+h}. \tag{4}$$

Equation (4) implies $q\to 0$ as $h\to 0$ if $d\leq 2$ and $T>0$.

Numerical calculations suggesting the absence of gauge glass ordering in two dimensions[4-8] have thus been justified rigorously. It should be noted that our result is that the lower critical dimension (lcd) is two or larger, not necessarily that the lcd is equal to two. In fact, numerical evidence shows that the lcd is close to three.[3-8]

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